E-20-655

A Stochastic Control Method For Hydropower Scheduling

by

Aris P. Georgakakos

Assistant Professor

Final Report

Georgia Power Company research Project POE-06608

August 1989

School of Civil Engineering
Georgia Institute of Technology
A Unit of the University System of Georgia
Atlanta, Georgia 30332

A Stochastic Control Method For Hydropower Scheduling

by

Aris P. Georgakakos

Assistant Professor

Final Report

Georgia Power Company Research Project POE–06608

August 1989

School of Civil Engineering

Georgia Institute of Technology

A Unit of the University System of Georgia

Atlanta, Georgia 30332

Contents

List of Figures | Page

List of Figures Cont'd | Page

List of Tables

Acknowledgements

The research work reported herein was sponsored by Georgia Power Company, Research Project POE-06608, and by the United States Geological Survey, Water Resources Act 1984, Grant No. 14-08-0001-G1297.

The author is grateful to Mr. Marty Meeks of Georgia Power Company for providing all necessary data and his valuable experience with respect to the operation of the Lloyd Shoals hydroelectric facility. The assistance of Mr. Thomas Barr, graduate student, in preparing some of the report figures is also gratefully acknowledged.

Disclaimer

The contents of this report were developed in part under a grant from the Department of the Interior, U. S. Geological Survey. However, those contents do not necessarily represent the views and policies of that agency, and you should not assume endorsement by the Federal Government.

1. Introduction

Optimal scheduling of hydropower operations is a process involving a plethora of complicating factors. At any given day, reservoir operators must skillfully balance upcoming inflow forecasts against available storage, turbine power, and discharge capacities to maximize energy generation. Hydropower is most valuable during the day's "peak" generation period, and, therefore, hydro-plants must generate as much energy as possible during the peak hours. Hydropower turbines should optimally operate at best efficiency, where a given release volume generates the most energy. However, at times of high flows, it pays to abandon best efficiency operation and "run" at full gate. During off-peak hours, energy is normally produced at a required minimum except when peak generation cannot maintain desirable reservoir levels. During such occasions, off-peak generation should be invoked as much as necessary. At times of extremely high flows, emergency flood gates may have to be considered, while during extreme droughts, power generation may have to cease.

This work researches and implements a new control method for the optimal short-term scheduling of hydropower systems. The method is based on a problem formulation which allows the application of stochastic control techniques. Such techniques have successfully been employed in long-term reservoir control (see, for instance, Wasimi and Kitanidis [1983], Marino and Loaiciga [1985], Georgakakos and Marks [1987], and Georgakakos [1989a,b]), while their application to short-term scheduling problems is at an early stage (see, Trezos and Yeh [1987]). The new model will be used for the day-to-day operations scheduling of the Lloyd Shoals hydroelectric project which is owned and operated by the Georgia Power Company.

This report includes five additional sections. Section 2 provides a general description of the Lloyd Shoals hydroelectric facility and dam. Section 3 compiles all data and relationships necessary for the development of the mathematical system model. Section 4 outlines the control model structure and provides the mathematical details of the algorithms used. Section 5 includes two parts. The first presents some computational experience with the new control model, and the second elaborates on the results from five simulation experiments. Section 6 summarizes the primary findings of this work and provides the basis for potential system improvements.

2. System Description

The subject of this study is the Lloyd Shoals Hydroelectric Project, which is owned and operated by the Georgia Power Company. As shown in Figure 1, the project is located on the Ocmulgee River approximately 45 miles southeast of Atlanta, Georgia. Also indicated in the figure are the operational hydrologic stations. None of the four major streams supplying Lake Jackson are gauged near their confluence with the lake.

Originally built and owned by the Central Georgia Power Company, Lloyd Shoals began producing electricity in 1911 with four 2400-kilowatt units. A fifth 2400-kilowatt unit was completed in 1916, and a sixth 2400-kilowatt unit was added a year later, bringing the plant's output to 14,400 kilowatts. Today, as a result of improvements made over the years, the Lloyd Shoals plant can generate 20,000 kilowatts.

From east to west the dam consists of an earth embankment section with a concrete core wall about 530 feet long, a concrete gravity structure including a 728.5–foot long overflow spillway, an intake section (198 feet), and a non–overflow section (143 feet). The crests of the earth embankment and the non–overflow sections are at elevations 542 and 540 respectively, and the top of the spillway flashboards are at elevation 530. The maximum dam height is about 100 feet.

The overflow spillway section consists of a 180–foot section on the east end and a 128.5–foot section on the west end with a crest elevation of 528. These two sections are equipped with two-foot high flashboards which trip slightly above elevation 530. The 420–foot middle section has a crest elevation of 525 feet and is equipped with five–foot high flashboards that also trip slightly above elevation 530. Included in the western section is a

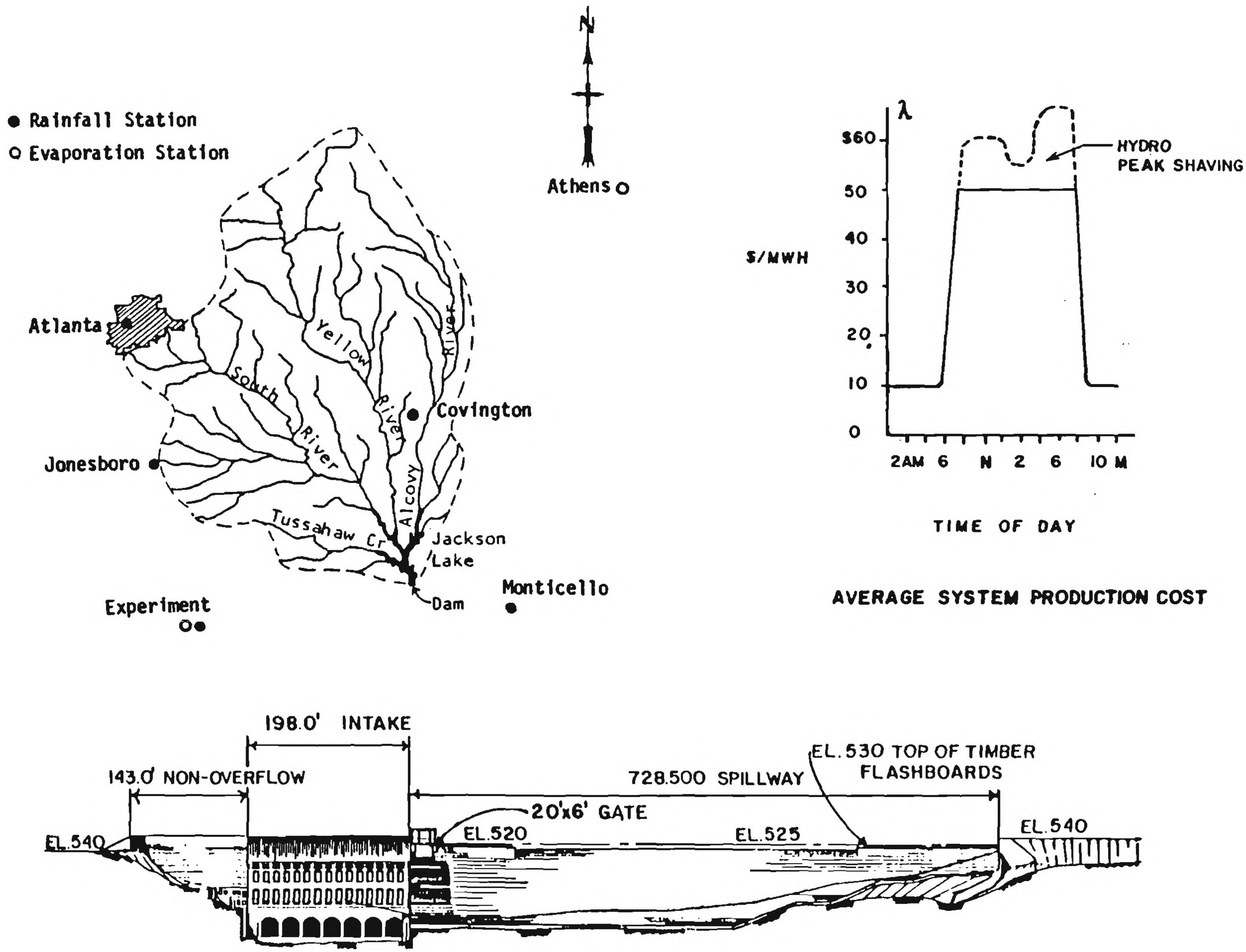


Figure 1: The Lloyd Shoals Hydroelectric Project

20-foot wide by 6.2-foot high gate with a sill at elevation 518. This gate is provided for reservoir regulation and trash release as well as supplemental spillway discharge.

The reservoir formed by the dam, Jackson Lake, has a surface area of 4750 acres at the normal pool elevation of 530 feet. The drainage area is approximately 1400 square miles.

The Georgia Power Company hydroelectric plants operate for the primary purpose of power generation. In general, flood control, water supply, or navigation are not operational objectives. The total hydroelectric capacity of the Georgia Power Company system is approximately 5.5 percent of the total system generating capacity. From an energy standpoint, the hydroelectric plants supply roughly 3 percent of Georgia's energy needs.

The daily demand on the system typically varies from a low during late night/early morning hours to a peak during the normal business hours. The annual maximum load generally occurs during the summer between 4 and 6 p.m. due to air conditioning needs. The peak generation period lasts approximately fourteen hours each week day. As the load increases from the late night/early morning minimum to the peak period, variable production costs also increase due to the use of fossil-fueled plants. The average system production cost at any time is termed the system lambda. Figure 1 additionally includes an example of a lambda curve. The values shown are for illustration only. Since the variable cost of a hydroelectric plant is essentially zero, they are operated when they are most effective—on the peak of the lambda curve. Consequently, Georgia Power Company's hydroelectric plants are "peaking" plants.

From the previous discussion, it is obvious that the operating objective at the Lloyd Shoals Project is to maximize the energy generated during the fourteen hour peak period Monday through Friday of each week. This requires maximizing the hydraulic head while avoiding overtopping/tripping the flashboards at elevation 530. Additional reservoir level constraints are that the level should not be lower than elevation 522 from September 1 to June 1 and elevation 528 the remainder of the year. The elevation 528 constraint occurs because Lake Jackson is used as a water source in extreme droughts to supply Georgia Power Company thermal plants located downstream. During off-peak periods a minimum generation level is desired. Normally this is 2000 kilowatts per hour; however, during dry periods, the plant's generation may be reduced to 500 kilowatts per hour or even down to zero to minimize the drawdown in the lake level. Finally, a minimum instantaneous downstream flow of 100 cfs must be maintained. This last requirement is usually satisfied by turbine wicked-gate leakage.

3. System Model

Scheduling reservoir operation requires models of the following key system elements: dynamics, operational constraints, and objectives.

System dynamics describe the system's response to various inputs and outputs and on a day-to-day basis can be modelled by:

$$s(k+1) = s(k) - u(k) - g(k) - \ell(k) - f(k) + w(k), \qquad (1)$$

$$k = 0,1,\ldots,N,$$

where $s(k)$ represents reservoir storage at the beginning of time period k; $u(k)$ represents turbine release during the scheduled generation hours; $g(k)$ is the release from the turbine assigned to meet the minimum generation requirement of 2000 KW per day; $\ell(k)$ is turbine leakage; $f(k)$ is flood gate or spillway outflow; $w(k)$ is net reservoir inflow; k is the time discretization interval (corresponding here to one day); and N is the length of the control horizon.

The turbine release volume $u(k)$ can be expressed as

$$u(k) = \left[\sum_{i=1}^{6} u_i(k)\right] t(k), \qquad (2)$$

where $u_i(k)$ represents the discharge of the i^{th} turbine and $t(k)$ is the scheduled power generation time during period k. These discharges depend upon the reservoir level and may correspond to best turbine efficiency, maximum power output, or some other operational mode.

The minimum generation release $g(k)$ is given by

$$g(k) = [24 - t(k)]\, u_j^{min}(k), \qquad (3)$$

where $u_j^{min}(k)$ is the discharge required to generate 2000 KW from the turbine designated for this purpose.

Turbine leakage is related to storage through

$$\ell_i(k) = \ell_i^{ref} \sqrt{(h[s(k)] - h_0) / h^{ref}}, \tag{4a}$$

$$\ell(k) = \sum_{i=1}^{6} \ell_i(k), \tag{4b}$$

where h_0 corresponds to the elevation of the turbine centerline; ℓ_i^{ref} are the leakage rates at some reference head h^{ref}, and $h[s(k)]$ is the reservoir's elevation-storage relationship (Georgia Power, 1988). The values of these parameters are reported below:

Table 1: Parameters of the Leakage Functions

h_0 = 446 feet	h^{ref} = 81.24 feet
Turbine	ℓ_i^{ref} [cfs]
1	37.2
2	76.9
3	27.5
4	37.9
5	39.6
6	45.9
Total	265.0

The elevation-storage relationship (Figure 2) was determined via regression analysis on actual elevation-storage data:

$$h[s(k)] = 443.81903 + 0.32732543\ s(k) - 0.79649653\times10^{-3}\ [s(k)]^2 + 21.076469\ \ell n[s(k)] - 1.7542414\ (\ell n[s(k)])^2 + 10.757541/s(k), \tag{5}$$

where h is obtained in feet when s(k) is expressed in 1000 acre-feet. Some regression statistics are reported in the following table.

Table 2: Statistics of the Elevation Storage Regression Equation

% of Variation Explained by Regression	99.9925
St. Deviation of Residuals [ft]	0.2183
Max. Pos. Deviation of Residuals [ft] (Predicted - Actual)	0.5893
Max. Neg. Deviation of Residuals [ft] (Predicted - Actual)	-0.3207

Flood gate outflow is also related to storage in a nonlinear fashion and is modelled as orifice or weir flow depending on whether or not the gate opening is submerged. The associated equations are as follows:

(i) If 518 feet $\leq h[s(k)] \leq$ 524.4 feet (orifice),

$$f(k) = \alpha_0 \; [L - \alpha_1 \; (h(s(k) - h_0)] \; [h[s(k)] - h_0]^{1.5} \; , \qquad (6a)$$

where α_0=3.1, L=20 feet, α_1=0.2, h_0=518 feet, h in feet, and f(k) in cfs.

(ii) If $h[s(k)] \geq$ 524.4 feet (weir),

$$f(k) = L \; d \; \beta_0 \; [\; 2 \; g \; (h[s(k)] - h_1)]^{0.5} \quad , \qquad (6b)$$

where d=6 feet, g=32.17 feet/sec^2, h_1=521 feet, h in feet, f(k) in cfs, and β_0 is a coefficient which depends on the water depth as follows (Brater and King, 1967):

Table 3: Weir Flow Coefficients

H = h(s(k)) - 521 [feet]	β_0
3.4	0.530
3.9	0.535
5.7	0.569
7.6	0.584
9.4	0.595
≥ 12.0	0.600

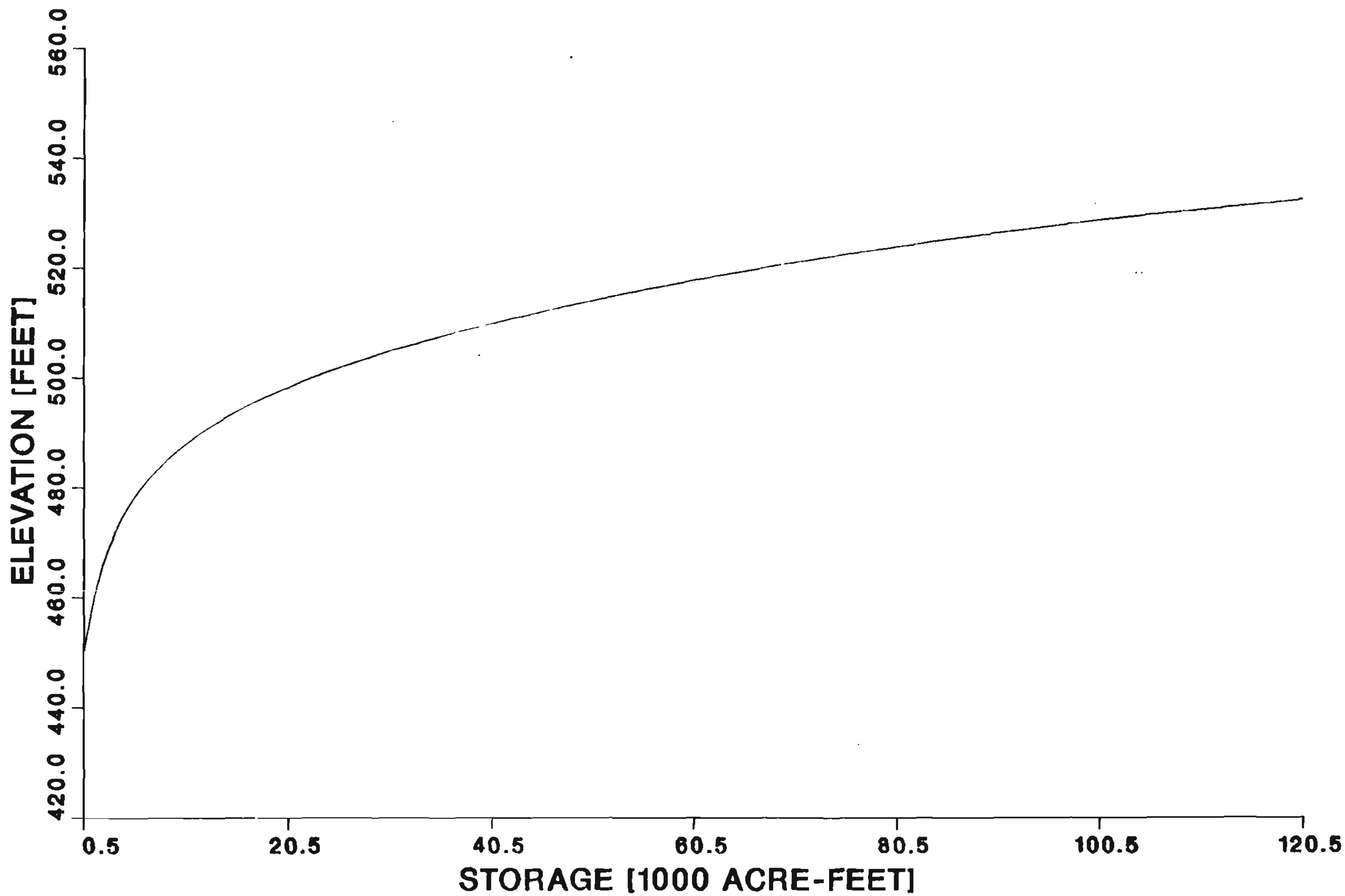


Figure 2: Elevation Versus Storage Relationship

Operational and physical constraints limit the variation of storage, release, and generation time to their feasible ranges:

$$s^{min}(k) \le s(k) \le s^{max}(k), \tag{7a}$$

$$u_i^{min}(k) \le u_i(k) \le u_i^{max}(k), \; i=1,2,\ldots,6, \tag{7b}$$

$$t^{min}(k) \le t(k) \le t^{max}(k), \tag{7c}$$

$$k = 0,1,\ldots,N,$$

where s^{min} and s^{max} correspond to the minimum and maximum allowable storages; u_i^{min} and u_i^{max} are the discharges corresponding to the minimum and maximum turbine power output; and t^{min} and t^{max} determine the hours of energy generation within a day. Based on the discussion in the previous section, s^{min} can be taken equal to 3218 $\times 10^6$ ft^3 (522 feet) from September 1st through June 1st and 4247 $\times 10^6$ ft^3 (528 feet) for the remainder of the year, and s^{max} is equal to 4660 $\times 10^6$ ft^3. The minimum power output for all turbines is equal to 500 KW, while the maximum power output equals 3200 KW for turbines 1 to 4, and 3400 KW for turbines 5 and 6. t^{min} is equal to zero, and t^{max} can be 14 or 24 hours on a week day depending on the operational mode (14 corresponds to the duration of the peak period), and 0 or 24 hours on Saturdays and Sundays.

Due to inflow uncertainty, constraints (7a) should be restated in a probabilistic format:

$$\text{Prob}[s(k) \le s^{min}(k)] \le \gamma^{min}(k), \tag{8a}$$

$$\text{Prob}[s(k) \ge s^{max}(k)] \le \gamma^{max}(k), \tag{8b}$$

where γ^{min} and γ^{max} are the probabilistic tolerance levels and $k=1,2,\ldots,N-1$.

The purpose of the Lloyd Shoals project is to maximize energy output. The power generation functions for each turbine were developed via regression analysis on simultaneous head, flow, and power measurements and have the following form (Georgia Power, 1988):

$$p_i[h_i^n, u_i] = a_{i,1} + a_{i,2}\, h_i^n + [b_{i,1} + b_{i,2}\, h_i^n]\, u_i + \\ + [c_{i,1} + c_{i,2}\, h_i^n]\, u_i^2, \qquad (9)$$

$$i = 1, 2, \ldots, 6,$$

where $p_i[\ ]$ is the power output (KW), h_i^n is the net hydraulic head (feet), u_i is the turbine discharge (cfs) and a,b, and c are regression coefficients. The values of these coefficients are reported below:

Table 4: Coefficients of the Turbine Power Functions

$a_{1,1}$ = - 1144.1307	$a_{1,2}$ = 5.153575
$b_{1,1}$ = 5.314086	$b_{1,2}$ = 0.0283214167
$c_{1,1}$ = - 0.004464149	$c_{1,2}$ = 0.0000172344
$a_{2,1}$ = - 1694.5590	$a_{2,2}$ = 5.0963333
$b_{2,1}$ = 5.511485	$b_{2,2}$ = 0.03296675
$c_{2,1}$ = - 0.00428132	$c_{2,2}$ = 0.000015404833
$a_{3,1}$ = - 1539.152	$a_{3,2}$ = 5.0631667
$b_{3,1}$ = 5.619713	$b_{3,2}$ = 0.03401
$c_{3,1}$ = - 0.004682089	$c_{3,2}$ = 0.00001684275
$a_{4,1}$ = - 1757.187	$a_{4,2}$ = 5.096
$b_{4,1}$ = 5.568577	$b_{4,2}$ = 0.033434
$c_{4,1}$ = - 0.004446496	$c_{4,2}$ = 0.000016007917
$a_{5,1}$ = - 2219.861	$a_{5,2}$ = 5.70675
$b_{5,1}$ = 6.171604	$b_{5,2}$ = 0.036911167
$c_{5,1}$ = - 0.004794143	$c_{5,2}$ = 0.000017251417
$a_{6,1}$ = - 1425.7394	$a_{6,2}$ = 4.7300083
$b_{6,1}$ = 5.099076	$b_{6,2}$ = 0.03046375
$c_{6,1}$ = - 0.003934244	$c_{6,2}$ = 0.0000141565

Figure 3 displays the power function of the first turbine.

The net hydraulic head is determined based on the reservoir forebay elevation, the tailwater elevation, and the frictional energy losses. For a total outflow Q, the tailwater elevation can be computed from (Figure 4)

$$Q \leq 3300 \text{ cfs} : t_w(Q) = 423.43666 + 2.8724141\, Q - 1.7019926\, Q^2 + $$
$$+ 0.69391531\, Q^3 - 0.09701342\, Q^4, \tag{10a}$$

$$Q \geq 3300 \text{ cfs} : t_w(Q) = \exp\{\, 6.0498451 + 0.75771347 \times 10^{-2}\, [\ell n(Q)]$$
$$+ 0.64189658 \times 10^{-3}\, [\ell n(Q)]^2 - 0.98717478 \times 10^{-3}\, [\ell n(Q)]^3$$
$$+ 0.28708173 \times 10^{-3}\, [\ell n(Q)]^4 \,\}, \tag{10b}$$

where t_w in obtained in feet when Q is expressed in 1000 cfs. Some statistics of the above regression equations are included in the following table:

Table 5: Statistics of the Tailwater Regression Equations

	$Q \leq 3300$ cfs	$Q \geq 3300$ cfs
% of Variation Explained by Regression	99.9603	99.9976
St. Deviation of Residuals [ft]	0.0246	0.0234
Max. Pos. Deviation of Residuals [ft] (Predicted - Actual)	0.0352	0.0352
Max. Neg. Deviation of Residuals [ft] (Predicted - Actual)	-0.0360	-0.0340

The frictional energy losses can be estimated from

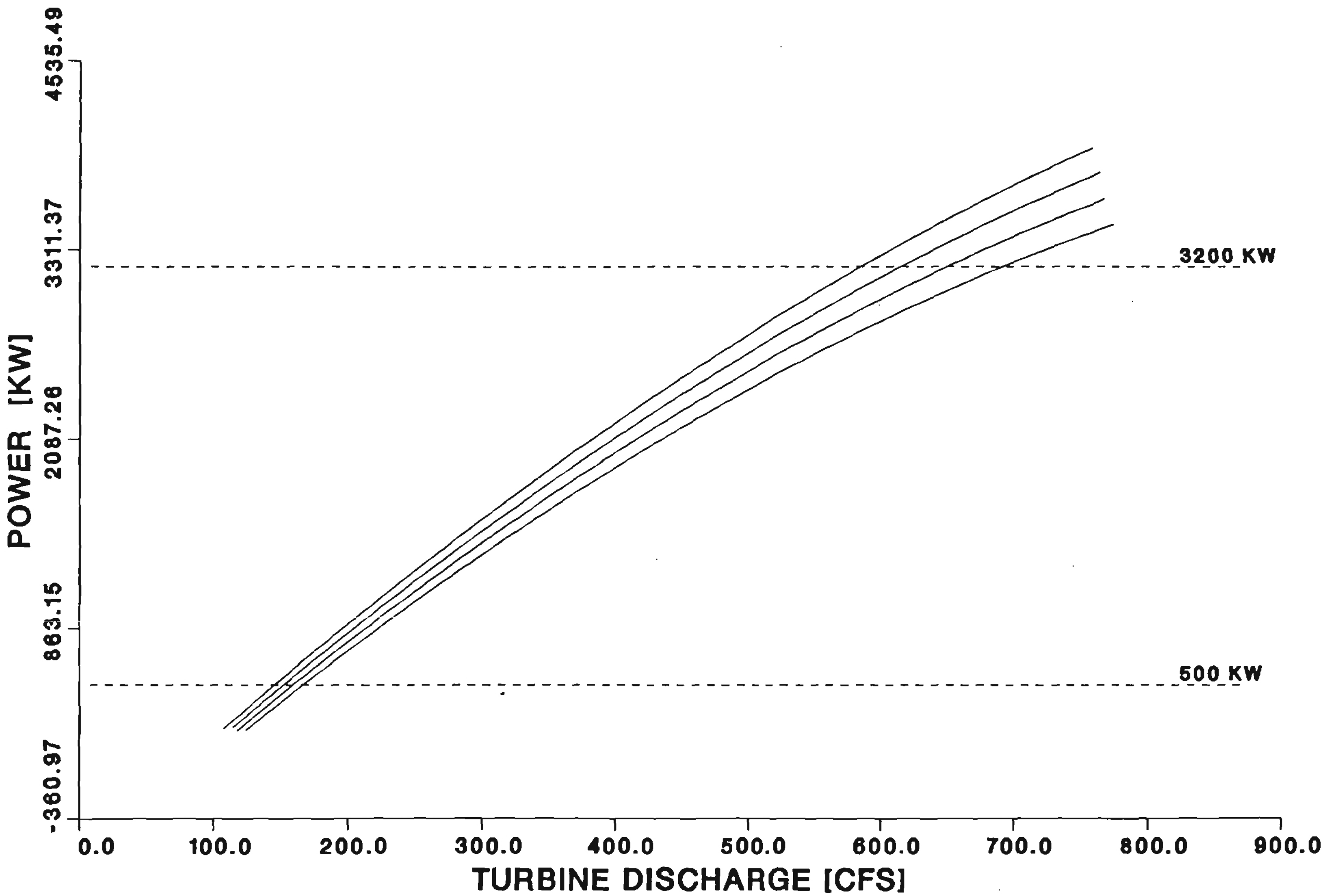
$$f_r(u_i) = 1.5\, [u_i/580]^2, \tag{11}$$

where f_r is obtained in feet when u_i is expressed in cfs (Figure 5, Georgia Power, 1988).

Lastly, the net hydraulic head is obtained as follows:

$$h_i^n = h[s(k)] - t_w(Q) - f_r(u_i). \tag{12}$$

It is noted that although the previous system model was motivated by the Lloyd Shoals hydroelectric project, it includes all elements characterizing any reservoir system expected to provide hydroelectric services. Furthermore, the control method presented in the next section is also generally applicable.



<u>Figure 3:</u> The Power Function of the First Turbine

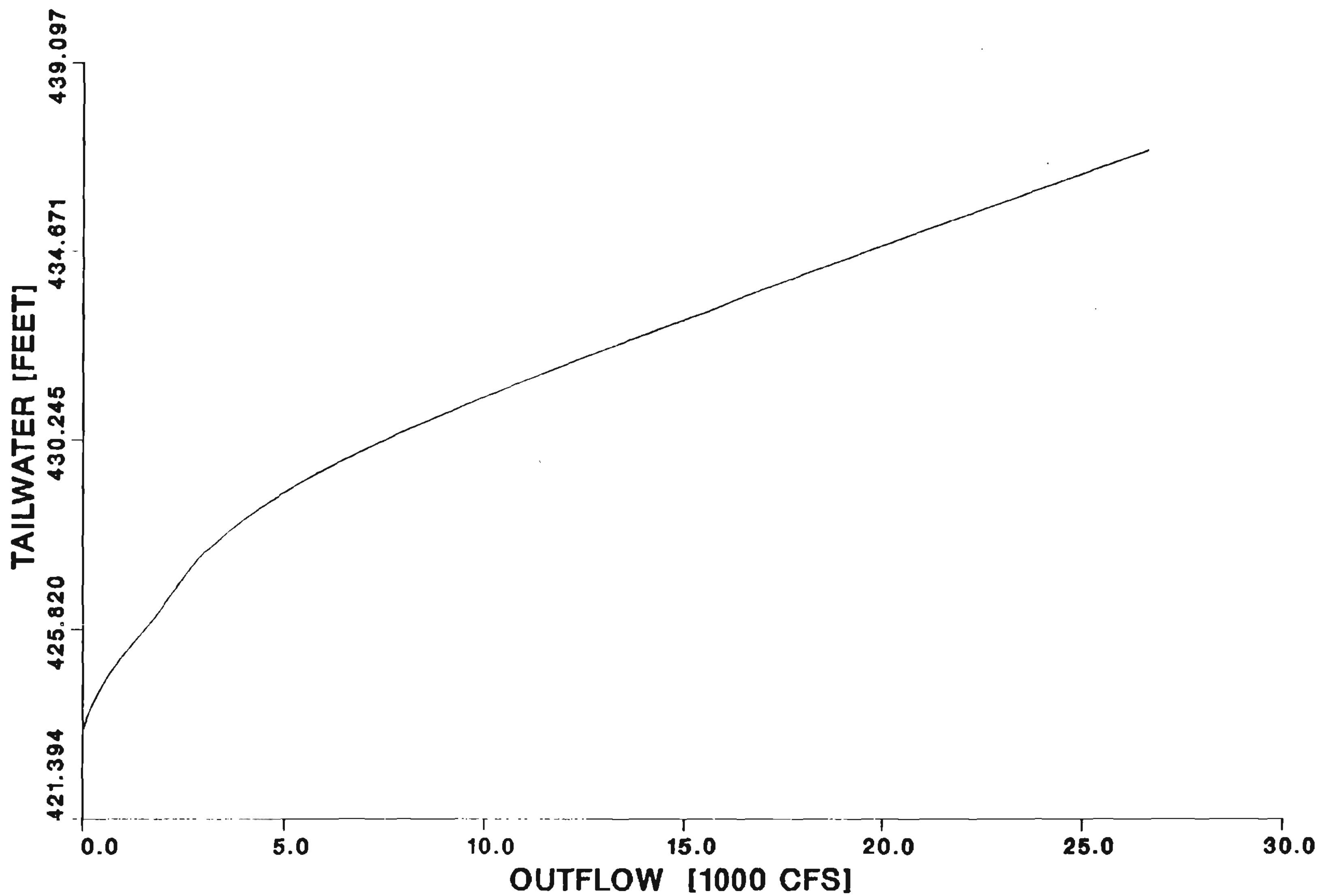


Figure 4: Tailwater Curve

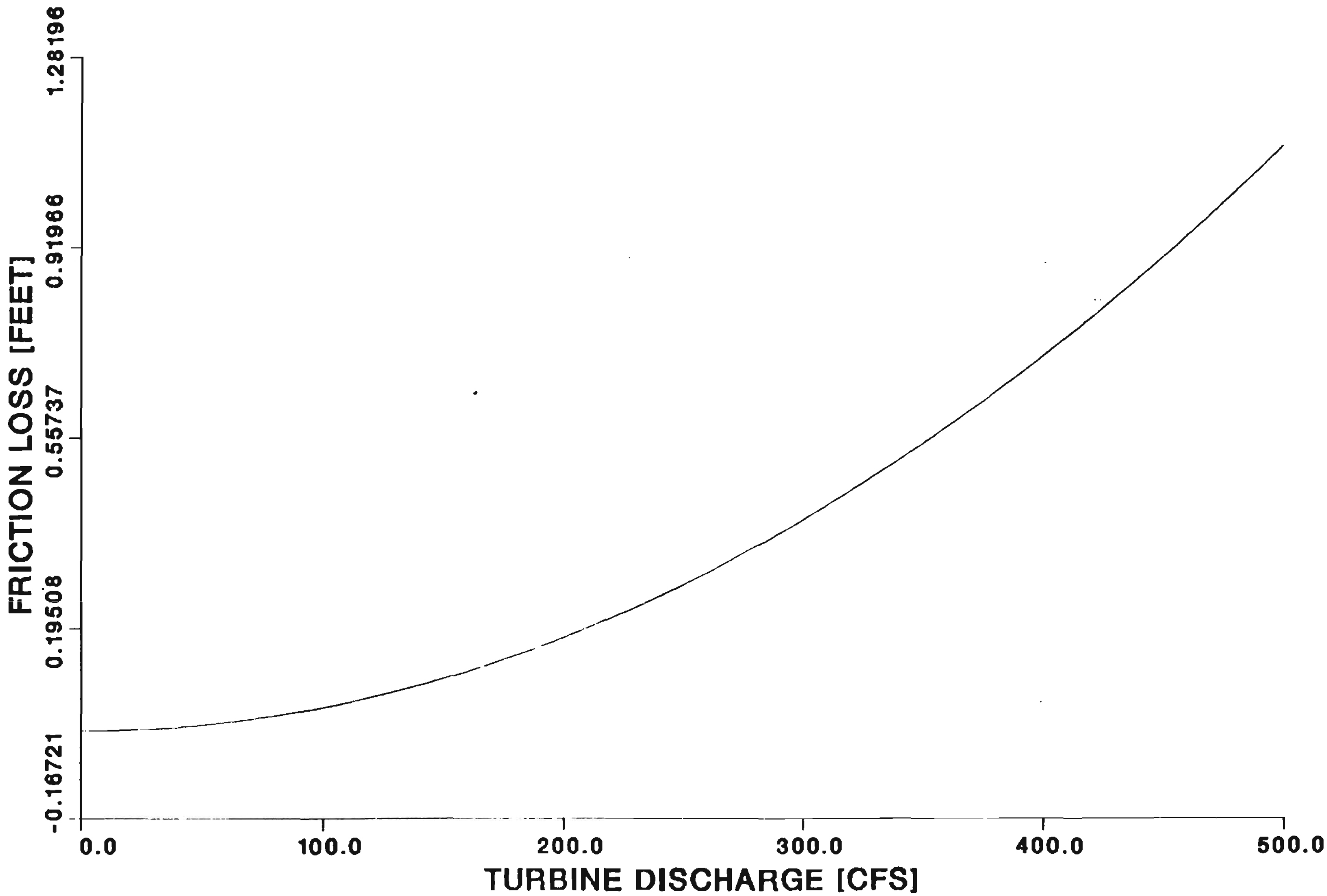


Figure 5: Hydraulic Frictional Losses

4. Control Model

4.1 Overview

The control model is a stochastic dynamic optimization scheme which determines optimal daily release and power generation schedules. This model is based on the Extended Linear Quadratic Gaussian (ELQG) reservoir control method (Georgakakos and Marks, 1987, Georgakakos, 1988a,b), but it also includes certain new enhancements which make it more suitable for hydropower systems.

The control model (Figure 6) includes five operational levels which are activated in the following sequential manner: The goal of the first level is to determine the optimal generation time schedule and associated discharges which maximize turbine efficiency during the peak generation periods. If a feasible solution is found here, the process terminates. The optimal power generation schedules for the next day are implemented, and the decision process is repeated at the beginning of the next day. If, on the other hand, some of the upper storage bounds are violated, indicating high inflows, the controller activates its second level.

The second level abandons best efficiency operation and attempts to find a solution with the turbines running fully open; that is, at maximum power. Generation times are again constrained within the peak period. If this level fails to bring the reservoir storage within its bounds, the controller invokes its third level.

The third level relaxes the peak period restriction and additionally allows for off-peak generation at maximum power. If a feasible solution cannot be found still, the controller activates its fourth level.

The fourth level operates the hydroelectric plant at maximum power for 24

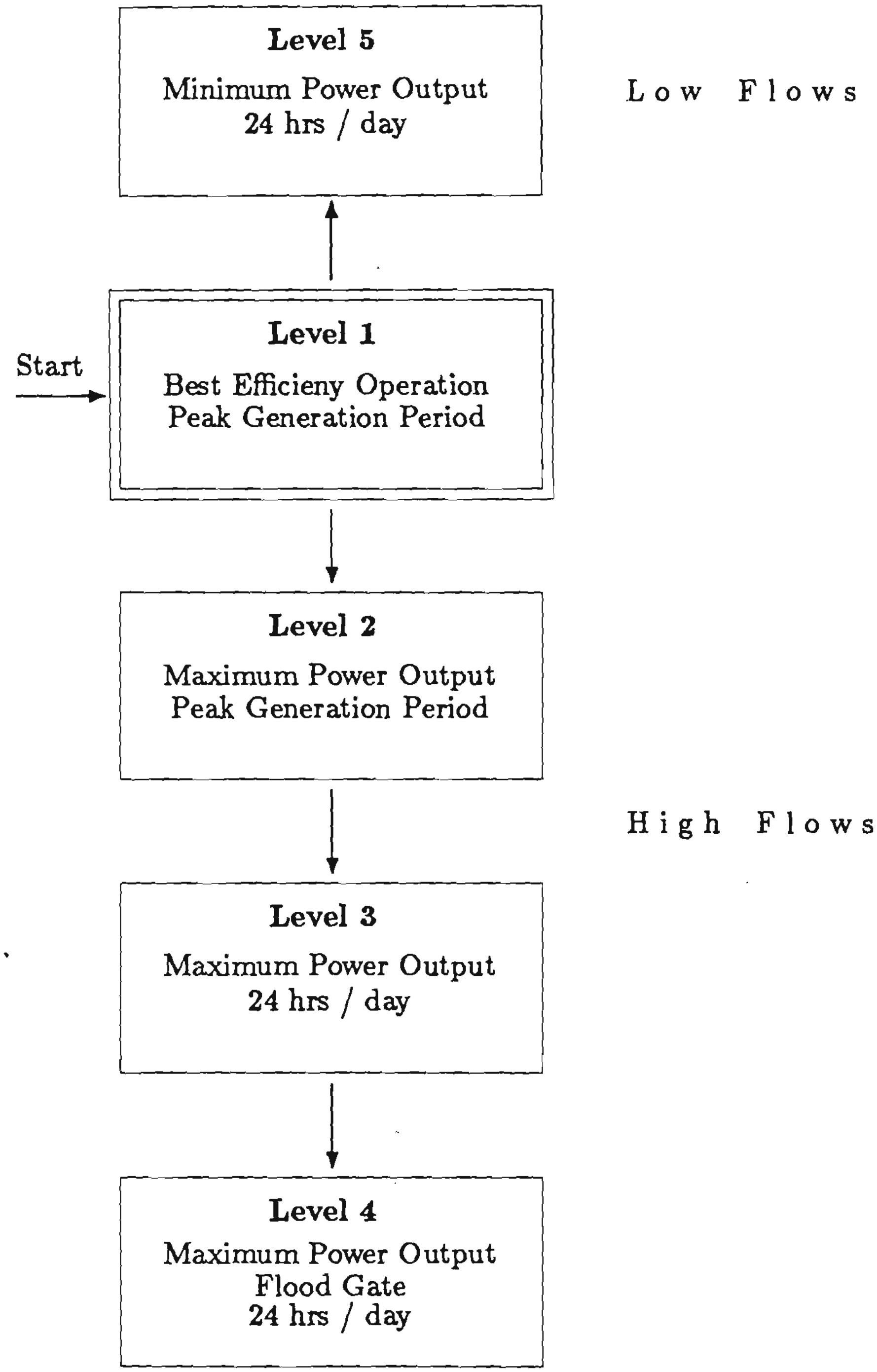


Figure 6: Control Model Structure

hours a day and additionally invokes the flood gate. The flood gate is the last operational control and, if it is unable to control the water levels from rising past the flash boards, the prescribed action is to run all available turbines and flood gate wide open.

If while in the first control level, the storage sequence violates its lower bounds (low flows), the controller actuates its fifth level. This level decreases the minimum generation requirement until the storage sequence becomes feasible. If this is not viable, the controller prescribes complete shut-down of the hydroelectric facility.

Each of the previous five levels, solves the real-time scheduling problem by utilizing a new, two-module, stochastic control procedure. A more detailed discussion of the functions of each control level and module follows next.

4.2 The First Control Level

The first control level initiates the control process by attempting to find optimal power generation and discharge schedules which maximize the efficiency of the turbines during the peak generation periods. In general, a streamflow forecasting model can be invoked at this stage to predict reservoir inflows over the control horizon. (Due to luck of adequate hydrologic data records, inflow forecasting can only be simulated here.) Subsequently, the control algorithm is called upon to solve the scheduling problem by iteratively activating two optimization modules (Figure 7).

The first optimization module (I) accepts the inflow forecasts from the forecasting model and estimates of the turbine discharges and leakage rates from the second module (II) and finds optimal power generation schedules (hours of generation within the peak generation period of each day.

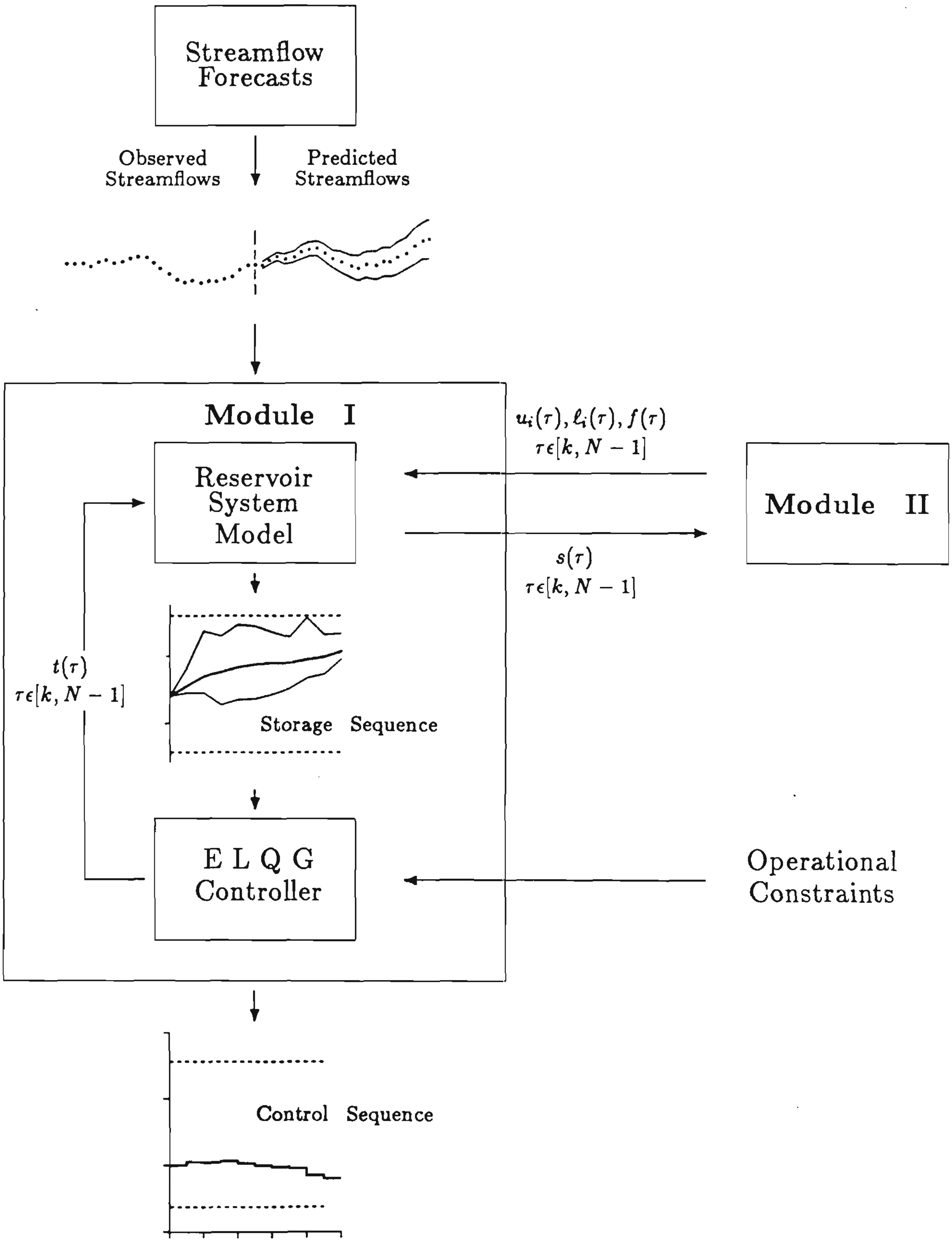


Figure 7: A Two-Module Control Method

The problem solved in this module is to find the optimal $\{t(k),\ k=0,\ldots,N-1\}$ sequence which minimizes

$$J = E\ \{ \sum_{k=0}^{N-1} \sum_{i=1}^{6} \xi_i(k)\ [\ P_i^{max} t_p - t(k)\ p_i[h_i^n, u_i]\ ]^2 + L[s(N),N]\ \} \tag{13}$$

subject to

(α) the storage dynamics:

$$s(k+1) = A(k)\ s(k) + B(k)\ t(k) + \omega(k) \tag{14}$$

where $A(k) = 1,$

$$B(k) = -\sum_{i=1}^{6} [\xi_i(k)\ u_i(k) + \psi_i(k)\ u_i^{min}(k) + (\xi_i(k) - \psi_i(k))\ \ell_i(k)],$$

$$\omega(k) = w(k) - 24 \sum_{i=1}^{6} [\psi_i(k)\ u_i^{min}(k) + (1 - \xi_i(k))\ \ell_i(k) + (\xi_i(k) - \psi_i(k))\ \ell_i(k)],$$

$$\xi_i(k) = \begin{cases} 1\ , & \text{if turbine i is operational during period k,} \\ 0\ , & \text{if turbine i is unavailable during period k,} \end{cases}$$

$$\psi_i(k) = \begin{cases} 1\ , & \text{if turbine i is designated to cover the minimum generation requirements during period k,} \\ 0\ , & \text{if turbine i will not cover the minimum generation requirements during period k;} \end{cases}$$

(β) the storage and generation time constraints:

$$\text{Prob}[s(k) \le s^{min}(k)] \le \gamma^{min}(k), \quad k=1,2,\ldots,N, \tag{15a}$$

$$\text{Prob}[s(k) \ge s^{max}(k)] \le \gamma^{max}(k), \quad k=1,2,\ldots,N, \tag{15b}$$

$$t^{min}(k) \le t(k) \le t^{max}(k), \quad k=0,1,\ldots,N-1. \tag{15c}$$

In the above index, t_p represents the target generation period (hours), and P_i^{max} is the maximum power output of turbine i. Equation (14) follows directly from the definitions given in the previous section. The values of ξ, ψ, s^{min}, s^{max}, γ^{min}, γ^{max}, t^{min}, and t^{max} are specified by the user. In this level,

$[t^{min}, t^{max}]$ represents the peak generation period of each day (8:00 am to 10:00 pm for Monday through Friday), and s^{min} and s^{max} are the minimum and maximum storages as discussed in the previous section. The terminal cost term $L[s(N),N]$ is a quadratic function with origin at the upper storage bound to reflect the long-term operation policy of maintaining high reservoir levels.

This is a stochastic control problem which can be solved via the ELQG control method (Georgakakos and Marks, 1987, Georgakakos, 1989a,b). For a detailed discussion of this method, the reader is referred to the previous citations. Herein the emphasis will be on the new method enhancements. The main difference between this and the more traditional reservoir control problem formulations (see, for instance, Georgakakos, 1989a) is that the optimization variables do not represent release volumes but rather power generation hours. As will be seen, this novelty leads to a more accurate description of the system's hydroelectric function.

In this level, the second control module (II) is commissioned to determine the discharge rates which maximize turbine efficiency over the interval $[0,t(k)]$ and satisfy the minimum generation requirements for the remainder of the day $[t(k),24]$. The efficiency e_i of the ith turbine is defined by

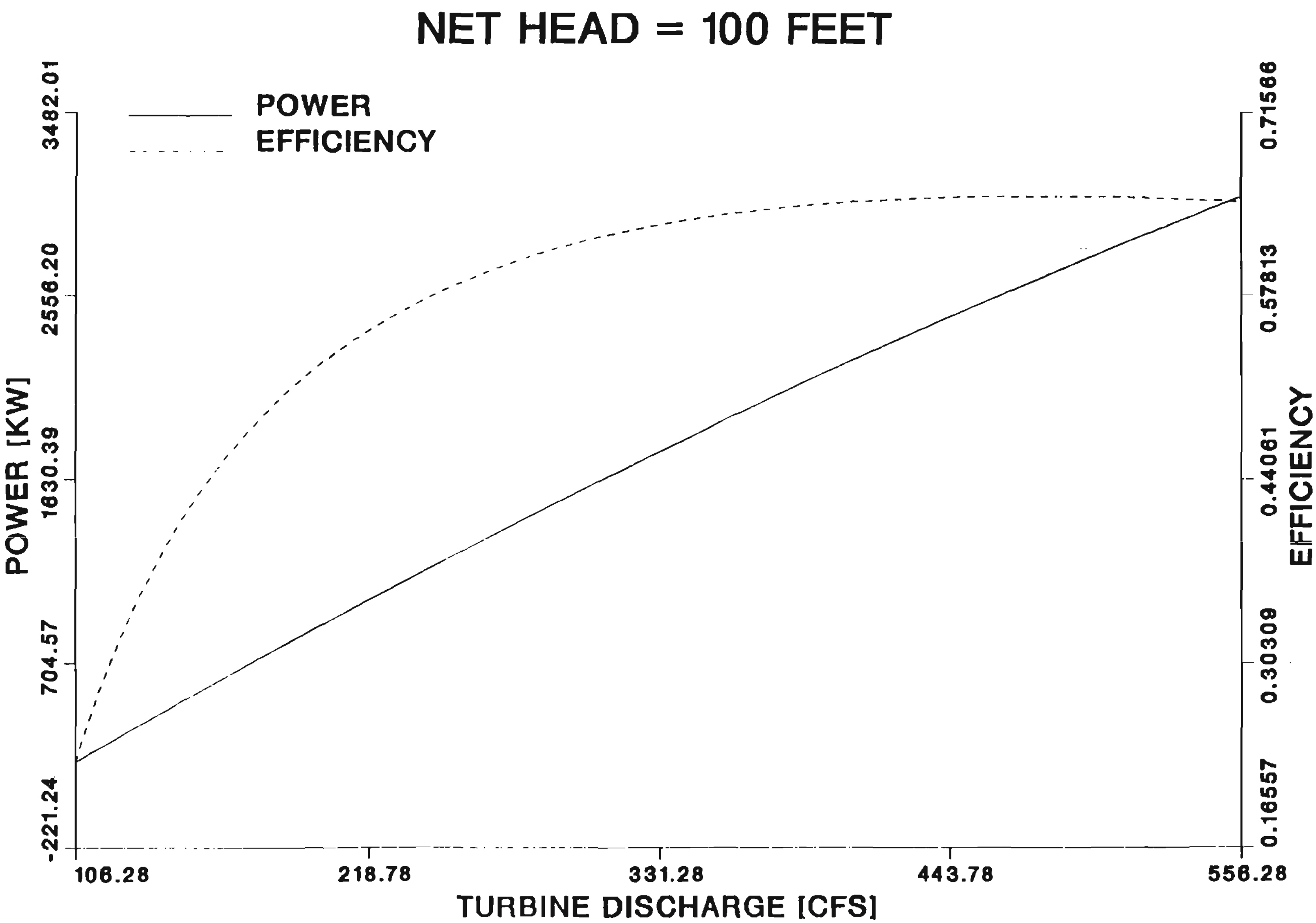
$$e_i = p_i \, / \, [\eta \; u_i \; h_i^n], \tag{16}$$

where p_i is the power generation function given by Equation (9), u_i is turbine discharge, h_i^n is the net hydraulic head, and η is a constant equal to 0.08465 when u_i is in cfs, h_i^n in feet, and p_i in KW. Figure 7 displays the power and efficiency curves of the first turbine for a net head of 100 feet.

Efficiency maximization is subject to power generation constraints,

$$P_i^{min} \le p_i \le P_i^{max}, \quad i=1,2,\ldots,6, \tag{17}$$

and involves all turbines due to the coupling introduced by the net hydraulic



<u>Figure 8:</u> Efficiency—Power Curves for the First Turbine

head. Thus, the problem to be solved in this module is to find the turbine discharges, $\{u_i,\ i=1,\ldots,6\}$, which maximize the efficiency of the available turbines, $\{e_i,\ i=1,\ldots,6\}$, subject to the power constraints (17). For this problem, reservoir storage is specified by the first control module. If the first module problem is stochastic, reservoir storage can be set equal to its expected value.

Substituting the power expression (9) in Equation (16), yields

$$e_i = \frac{1}{\eta\, u_i}\left[a_{i,1} + \frac{a_{i,2}}{h_i^n}\right] + \frac{1}{\eta}\left[b_{i,1} + \frac{b_{i,2}}{h_i^n}\right] + \frac{u_i}{\eta}\left[c_{i,1} + \frac{c_{i,2}}{h_i^n}\right], \tag{18}$$

where h_i^n depends on the discharge u_i through Equation (12).

Taking the first and second derivatives of the above expression with respect to u_i is a tedious task, but it establishes that $\partial^2 e_i/\partial u_i^2$ is negative and therefore demonstrates that e_i is a concave function of u_i. Thus, the optimal u_i's are those which set the first derivative of (18) equal to zero or, if this happens outside the feasible ranges defined by (16), those which correspond to the exceeded power bound. These optimal values are obtained here by the following algorithm:

<u>Algorithm IIα:</u> Specification of Turbine Discharge Rates for $[0,t(k)]$

1. Set $\{u_i(k),\ i=1,\ldots,6\}$ equal to some initial values.

2. Determine the forebay elevation $H = h[\bar{s}(k)]$, where $\bar{s}(k)$ is the mean storage value specified by the first control module.

3. Determine the total reservoir outflow rate:

$$Q = \sum_{i=1}^{6} [\xi i(k)\ u_i(k) + (1 - \xi_i(k))\ \ell_i(k)], \tag{19}$$

where $\ell_i(k)$ is the leakage rate given by Equation (4a).

4. Compute the net hydraulic head for each turbine:

$$h_i^n = H - t_w(Q) - f_r[u_i(k)],\ i=1,\ldots,6, \tag{20}$$

where $f_r[\ ]$ is the frictional loss function (Eq. (11)).

5. Determine the discharge rates $\bar{u}_i(k)$ which maximize turbine efficiency (Eq. (18)). Assuming that h^n are constant, these discharges can be analytically computed by taking the derivative of Equation (18) and setting it equal to zero. The result is the following:

$$\bar{u}_i(k) = \sqrt{\frac{a_{i,1} + \dfrac{a_{i,2}}{h_i^n}}{c_{i,1} + \dfrac{c_{i,2}}{h_i^n}}}\ , \quad i=1,\ldots,6, \tag{21}$$

6. Compute the power, $p_i[h^n,\bar{u}_i]$, i=1,...,6, associated with the previous net heads and discharge rates from Equation (9).

$$\text{If} \quad p_i^{min} > p_i[h^n,\bar{u}_i] \quad \text{or} \quad p_i[h^n,\bar{u}_i] > p_i^{max},$$

compute the discharge rates that correspond to the exceeded power bound. These discharge rates can be computed by setting the left-hand side of Equation (9) equal to the exceeded power bound and solving the resulting quadratic equation with respect to u_i. The solution is obtained from

$$\bar{u}_i(k) = \frac{-B_i + \sqrt{B_i^2 - 4\,A_i\,C_i}}{2\,A_i}\ , \tag{22}$$

where $A_i = c_{i,1} + c_{i,2}\, h_i^n,$

$B_i = b_{i,1} + b_{i,2}\, h_i^n,$

$Ci = a_{i,1} + a_{i,2}\, h_i^n - P^{bound},$

and the values of $a_{i,1}$, $a_{i,2}$, $b_{i,1}$, $b_{i,2}$, $c_{i,1}$, and $c_{i,2}$ are given in Table 4.

7. Update the total reservoir outflow rate,

$$\bar{Q} = \sum_1^6 [\xi_i(k)\ \bar{u}_i(k) + (1 - \xi_i(k))\ \ell_i(k)], \tag{23}$$

and compute the difference $D = [\tilde{Q} - Q]$. If $|D| \leq \epsilon$, terminate; otherwise repeat Steps 4 through 7. The value of ϵ controls the accuracy of the solution and can be set, for instance, equal to 1 cfs.

The rational behind this algorithm is that turbine efficiency and power output are primarily controlled by turbine discharge while net head adjustments from iteration to iteration are relatively small. As a result, this optimization scheme is characterized by fast convergence rate requiring about 2 to 3 iterations to convergence.

The second task of the second module is to determine the discharge rate $u_i^{min}(k)$ of the ith turbine which is designated to cover the minimum generation requirement of 2,000 KW over the remainder of the day $[t(k),24]$. These computations can also be organized in a similar algorithmic manner.

<u>Algorithm IIβ:</u> Specification of Turbine Discharge Rates for $[t(k),24]$

1. Set $u_i^{min}(k)$ equal to some initial value.

2. Determine the forebay elevation $H = h[\bar{s}(k)]$, where $\bar{s}(k)$ is the mean storage value specified by the first control module.

3. Determine the total reservoir outflow rate:

$$Q = u_i^{min}(k) + \sum_{j=1}^{6} [(1 - \psi_j(k)) \ \ell_j(k)], \tag{28}$$

where $\ell_j(k)$ is the leakage rate given by Equation (4a).

4. Compute the net hydraulic head for the ith turbine:

$$h_i^n = H - t_w(Q) - f_r[u_i^{min}(k)], \tag{29}$$

where $f_r[\ ]$ is the frictional loss function (Eq. (11)).

5. Compute the discharge rates that correspond to the minimum generation requirement P^{mg} :

$$\bar{u}_i^{min}(k) = \frac{- B_i + \sqrt{B_i^2 - 4 A_i C_i}}{2 A_i}, \tag{30}$$

where $A_i = c_{i,1} + c_{i,2}\, h_i^n,$

$$B_i = b_{i,1} + b_{i,2}\, h_i^n,$$

$$C_i = a_{i,1} + a_{i,2}\, h_i^n - P^{mg} \quad ,$$

and the values of $a_{i,1}$, $a_{i,2}$, $b_{i,1}$, $b_{i,2}$, $c_{i,1}$, and $c_{i,2}$ are given in Table 4.

6. Update the total reservoir outflow rate,

$$\bar{Q} = \bar{u}_i^{min}(k) + \sum_{j=1}^{6} [(1 - \psi_j(k))\; \ell_j(k)], \tag{31}$$

and compute the difference $D = [\bar{Q} - Q]$. If $|D| \leq \epsilon$, terminate; otherwise repeat Steps 4 through 6. Again, the value of ϵ controls the accuracy of the solution and can be set equal to 1 cfs.

The previous tasks of the second control module are to be performed for all time periods k of the control horizon [0,N-1]. Thus, the second control module accepts values of the mean storage trajectory from the solution of the first module problem and generates best efficiency and minimum generation discharge rates. These discharges are fed back and help update the first module solution, with this exchange continuing until convergence. At the completion of this process, the storage trajectory is examined for possible constraint violations. If storage constraints (8) are not violated, the model terminates and the first day's optimal generation schedule is implemented. If upper storage constraints are violated (probability of exceedance is higher than the specified tolerance γ^{max}, indicating flood condition), the controller activates its second level. If, on the other hand, lower constraints cannot be satisfied (drought condition), the controller activates its fifth level.

4.3 The Second Control Level

The second level is activated when best efficiency discharges and the peak generation time of 14 hours daily excluding weekends cannot lower reservoir storage within its feasible range. The purpose here is to investigate whether maximizing power output within the peak period will prevent storage constraint violations. The solution is again obtained using the two-module control scheme described in the previous section with the following specifications:

Module I: As in the first level.

Module II: Step 5 of Algorithm IIα for the computation of the best efficiency discharge rates is replaced by Step 6 which is now performed for all available turbines.

If this level's solution is feasible, the model terminates. Otherwise, it invokes the third level.

4.4 The Third Control Level

The third level lifts the peak generation period restriction and allows for up to 24-hour daily generation time for both week-days and weekends. The solution is obtained as in the previous levels with the following specifications:

Module I: The values of the parameters t_p and $\{t^{max}(k), k=0,\ldots,N-1\}$ in Index (13) and Constraints (15c) are set equal to 24 hours.

Module II: As in the second control level.

As before, if the solution is feasible, the model terminates; otherwise, it activates the fourth level.

4.5 The Fourth Control Level

The fourth level is activated when turbine discharge rates at maximum power output are inadequate to prevent reservoir levels from exceeding the flash board tripping threshold of 530 feet. The last operational control is to force reservoir storage within the permissible bounds by utilizing the flood gate. Generation times are now fixed to 24 hours, and turbine discharges correspond to the highest allowable power output. The problem here is to find the necessary flood gate operation schedules which produce feasible reservoir storages. The solution can again be found by using the two-module control scheme introduced earlier in the following manner:

Module I: The problem solved in this module can be stated as follows: Determine the optimal flood gate operation schedule $\{t_f(k),\ k=0,1,\ldots,N-1\}$ which minimizes

$$J = E\ \{ \sum_{k=0}^{N-1} [s(k) - s^*(k)]^2 + [s(N) - s^*(N)]^2 \} \tag{32}$$

subject to

(α) the storage dynamics:

$$s(k+1) = A(k)\ s(k) + B(k)\ t_f(k) + \omega(k) \tag{33}$$

where $A(k) = 1$,

$B(k) = f(k)$ (flood gate outflow rate as computed by Eq. (6))

$$\omega(k) = w(k) - 24 \sum_{i=1}^{6} [\xi_i(k)\ u_i(k) + (1 - \xi_i(k)\ \ell_i(k)]$$

$$\xi_i(k) = \begin{cases} 1\ , \text{ if turbine i is operational during period k,} \\ 0\ , \text{ if turbine i is unavailable during period k,} \end{cases}$$

(β) the storage and generation time constraints:

$$\text{Prob}[s(k) \le s^{min}(k)] \le \gamma^{min}(k),\quad k=1,2,\ldots,N, \tag{34a}$$

$$\text{Prob}[s(k) \geq s^{max}(k)] \leq \gamma^{max}(k), \quad k=1,2,\ldots,N, \tag{34b}$$

$$0 \leq t_f(k) \leq 24, \quad k=0,1,\ldots,N-1. \tag{34c}$$

In the above formulation, $s^*(k)$, $k=1,..,N$, can be set equal to the upper storage bounds, the purpose being to determine the necessary flood gate releases which will satisfy the violated storage constraints. As before, the solution of this stochastic control problem may be obtained by the ELQG control method.

Module II: The purpose of this module is simply to determine the discharge rates which maximize power output. The procedure is similar to the one presented in the third level with the exception of Steps 3 and 7, in Algorithm IIα, where the total reservoir outflows should be computed by

$$Q = \sum_{i=1}^{6} [\xi i(k)\ u_i(k) + (1 - \xi_i(k))\ \ell_i(k)] + f(k), \tag{35}$$

$$\bar{Q} = \sum_{i=1}^{6} [\xi i(k)\ \bar{u}_i(k) + (1 - \xi_i(k))\ \ell_i(k)] + f(k). \tag{36}$$

Flood gate release is the last control that can be exercised by the operator and even if reservoir storage cannot be sufficiently lowered, the model terminates suggesting that all turbines and the flood gate be operated fully open 24 hours a day. The reason for letting water through the flood gate is to keep reservoir storage from exceeding the level of 530 feet. As discussed in Section 2, when water level rises above this threshold, the flash boards trip and cannot be repositioned until water level falls below 528 feet, causing a substantial loss of hydraulic head and water volume.

4.6 The Fifth Control Level

Levels 2, 3, and 4 are sequentially invoked depending on the severity of the upcoming flood. The fifth level is invoked during droughts when the optimal generation times are zero, and certain lower storage constraints are still violated. This level simply reduces the initially specified 2000 KW minimum generation requirements by 500 KW at a time, and it terminates when storage becomes feasible. The solution is obtained by following the procedures outlined in the first level; the only difference is that Algorithm IIβ is implemented with reduced values of the minimum generation P^{mg}. These reductions amount to 500 KW each time. If P^{mg} is reduced to 500 KW without attaining storage feasibility, the controller prescribes that the power plant be completely shut down.

5. Case Studies

5.1 Control Experiments

This section presents some computational experience with the control scheme presented earlier. The purpose here is (1) to provide engineering insight to this method's performance in real time decision operations and (2) to familiarize the user with the program's result presentation format.

For all three control experiments to be presented, the reservoir is initially assumed to have a storage of 4,000 million cubic feet and the control horizon is taken equal to 14 days. The lower and upper storage bounds are equal to 3,118 and 4,660 million cubic feet respectively (corresponding to 522 and 530 feet reservoir elevations). The starting date is Friday, January 1st, 1988. Apart from maximizing peak energy output, the controller also attempts to maintain end-of-horizon storage as high as possible. The tolerance levels γ are set equal to 2.5%. The objective of the control experiments is to maximize peak power generation under three different hypothetical inflow forecast scenarios. The inflow forecast statistics are shown on Figure 9. The forecast probability distributions are assumed to be lognormal with mean and standard deviation as indicated. Table 6 presents the optimal schedules for the first experiment. (Results are included for the first ten days of the control horizon.) For each period (day) the following quantities are reported: turbine discharge and power output during the daily generation time; turbine leakage for the non-generation period; power generation time; mean end-of-day storage and reservoir level; minimum generation output, designated turbine, and discharge; and flood gate discharge and operation time. Figure 10 portrays the optimal generation time and storage sequences. The dashed lines delineate the associated bounds; the three lines in the storage graph represent the mean and the 95% probability band about the mean. As can be seen from these results, the controller finds the optimal solution while in the first operational level. Namely, turbines "run" at best efficiency for a portion of the 14-hour peak generation period. The second, third, ninth, and tenth days, corresponding to Saturdays and Sundays, have no peak generation periods. For Turbines 2, 4, and 5 best efficiency generation implies maximum power output.

Table 7 and Figure 11 summarize the results of the second computational experiment with intermediate flow statistics. The optimal sequences are now found with the system in the third level of operation where all six turbines "run" at full gate for up to 24 hours a day including weekends.

Lastly, Table 8 and Figure 12 report the results of the third computational experiment. Anticipating high flows, the controller resorts to the fourth level where the flood gate may be opened to prevent overtopping of the flash boards. Although overtopping may happen 14 days into the future, the controller indicates that the flood gate must be operated from the first day (3.48 hours) if the associated risk is to be kept lower than or equal to 2.5%.

The previous experiments were performed on a CYBER 180/990 digital computer. Each experiment required approximately 2 CPU seconds. High computational efficiency is a distinctive characteristic of this approach and is due to the excellent convergence properties of the control algorithms developed. As a result, microcomputer implementations are also feasible.

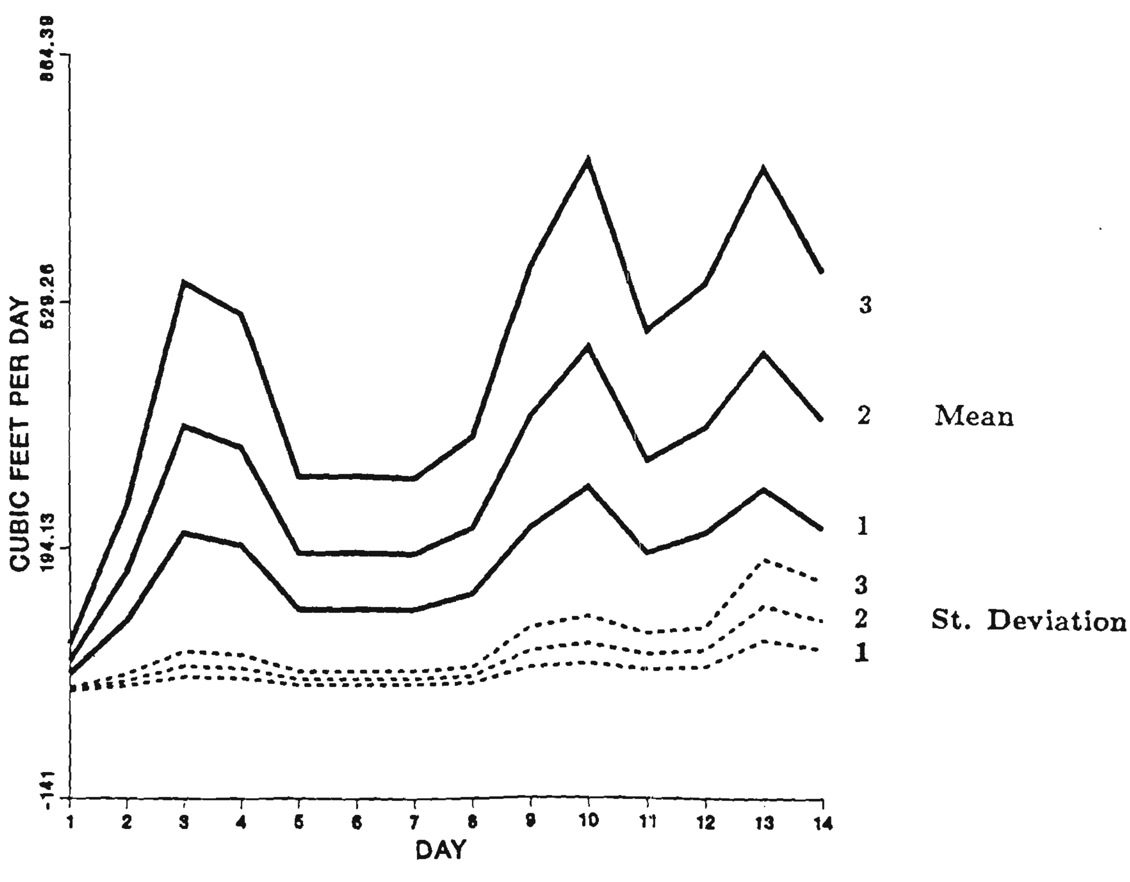


Figure 9: Inflow Forecast Statistics

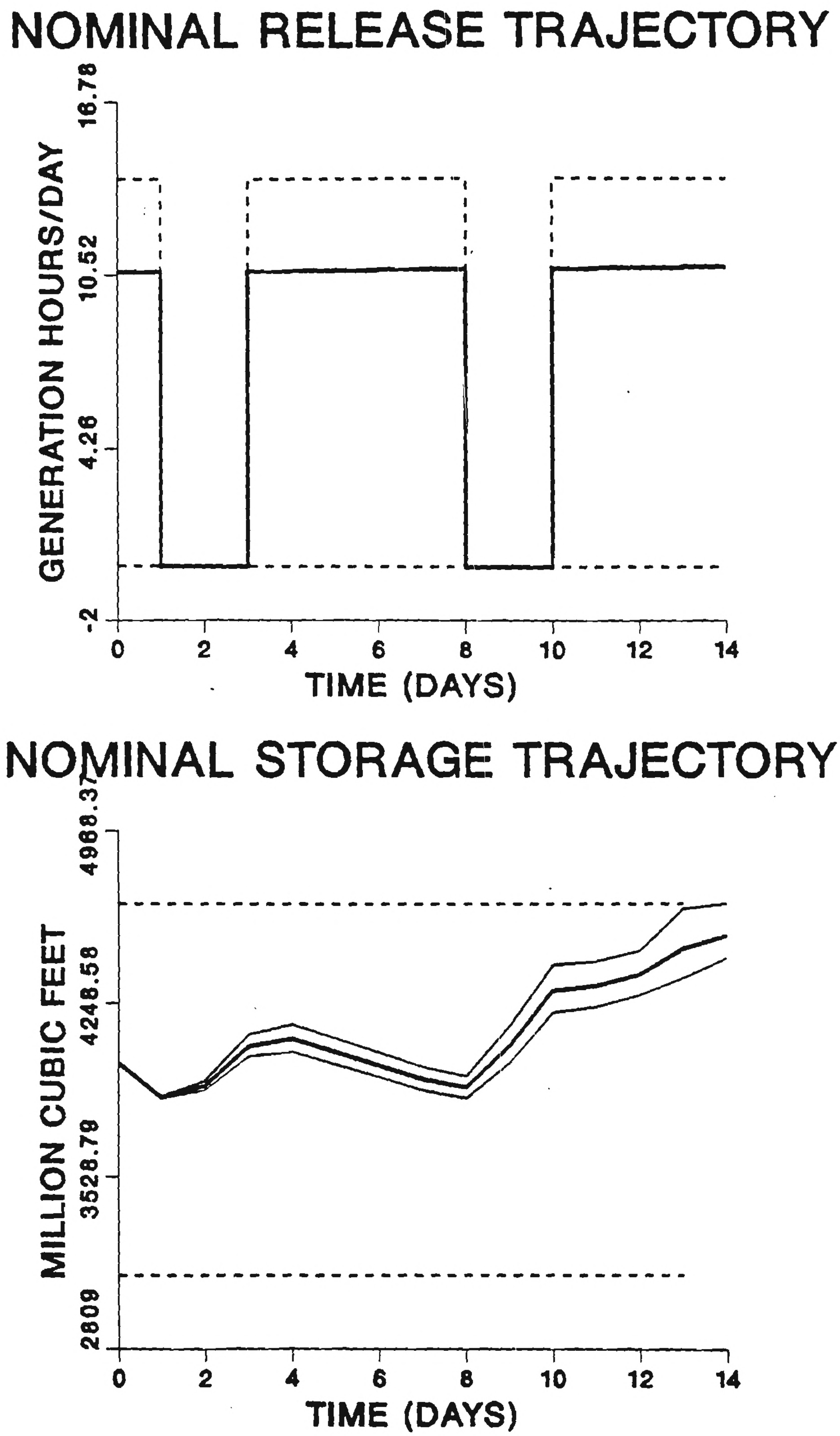


Figure 10: Optimal Sequences for Experiment #1

Table 6: Optimal Schedules for Experiment #1

OPTIMAL RELEASE AND POWER GENERATION SCHEDULES
STARTING DATE: 1/1/1988

PERIOD 1

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	480.06	634.94	586.36	643.23	660.35	614.44
POWER (KW):	2595.39	3200.00	3135.85	3200.00	3400.00	3012.80
LEAKAGE (CFS)	37.08	76.65	27.41	37.77	39.47	45.75

GENERATION TIME: 10.65 HRS
FORECASTED END-OF-PERIOD STORAGE: 3857.75 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.92 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 364.18 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 2

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	0.00	0.00	0.00	0.00	0.00	0.00
POWER (KW):	0.00	0.00	0.00	0.00	0.00	0.00
LEAKAGE (CFS)	36.90	76.27	27.27	37.59	39.28	45.52

GENERATION TIME: 0.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3904.02 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.17 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 366.47 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 3

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	0.00	0.00	0.00	0.00	0.00	0.00
POWER (KW):	0.00	0.00	0.00	0.00	0.00	0.00
LEAKAGE (CFS)	36.96	76.39	27.32	37.65	39.34	45.60

GENERATION TIME: 0.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 4068.52 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 527.07 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 365.71 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 4

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	479.90	632.52	586.44	640.69	657.74	614.51
POWER (KW):	2603.03	3200.00	3147.80	3200.00	3400.00	3023.94
LEAKAGE (CFS)	37.16	76.82	27.47	37.86	39.56	45.85

GENERATION TIME: 10.68 HRS
FORECASTED END-OF-PERIOD STORAGE: 4099.93 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 527.24 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 363.12 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 5

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	479.82	631.44	586.47	639.56	656.58	614.54
POWER (KW):	2606.48	3200.00	3153.21	3200.00	3400.00	3028.99
LEAKAGE (CFS)	37.20	76.90	27.50	37.90	39.60	45.90

GENERATION TIME: 10.70 HRS
FORECASTED END-OF-PERIOD STORAGE: 4045.68 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.95 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 362.64 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 6

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	479.95	633.32	586.41	641.53	658.60	614.49
POWER (KW):	2600.50	3200.00	3143.84	3200.00	3400.00	3020.25
LEAKAGE (CFS)	37.13	76.76	27.45	37.83	39.53	45.82

GENERATION TIME: 10.72 HRS
FORECASTED END-OF-PERIOD STORAGE: 3990.40 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.65 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 363.47 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 7

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	480.09	635.28	586.35	643.59	660.72	614.43
POWER (KW):	2594.31	3200.00	3134.16	3200.00	3400.00	3011.22
LEAKAGE (CFS)	37.06	76.62	27.40	37.76	39.46	45.73

GENERATION TIME: 10.75 HRS
FORECASTED END-OF-PERIOD STORAGE: 3933.54 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.34 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 364.33 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 8

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	480.23	637.35	586.29	645.76	662.94	614.36
POWER (KW):	2587.85	3200.00	3124.07	3200.00	3400.00	3001.80
LEAKAGE (CFS)	36.99	76.47	27.35	37.69	39.38	45.64

GENERATION TIME: 10.78 HRS
FORECASTED END-OF-PERIOD STORAGE: 3897.91 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.14 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 365.24 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 9

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	0.00	0.00	0.00	0.00	0.00	0.00
POWER (KW):	0.00	0.00	0.00	0.00	0.00	0.00
LEAKAGE (CFS)	36.95	76.38	27.31	37.64	39.33	45.59

GENERATION TIME: 0.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 4069.93 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 527.08 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 365.81 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 10

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	0.00	0.00	0.00	0.00	0.00	0.00
POWER (KW):	0.00	0.00	0.00	0.00	0.00	0.00
LEAKAGE (CFS)	37.16	76.82	27.47	37.86	39.56	45.85

GENERATION TIME: 0.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 4298.07 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 528.26 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 363.10 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

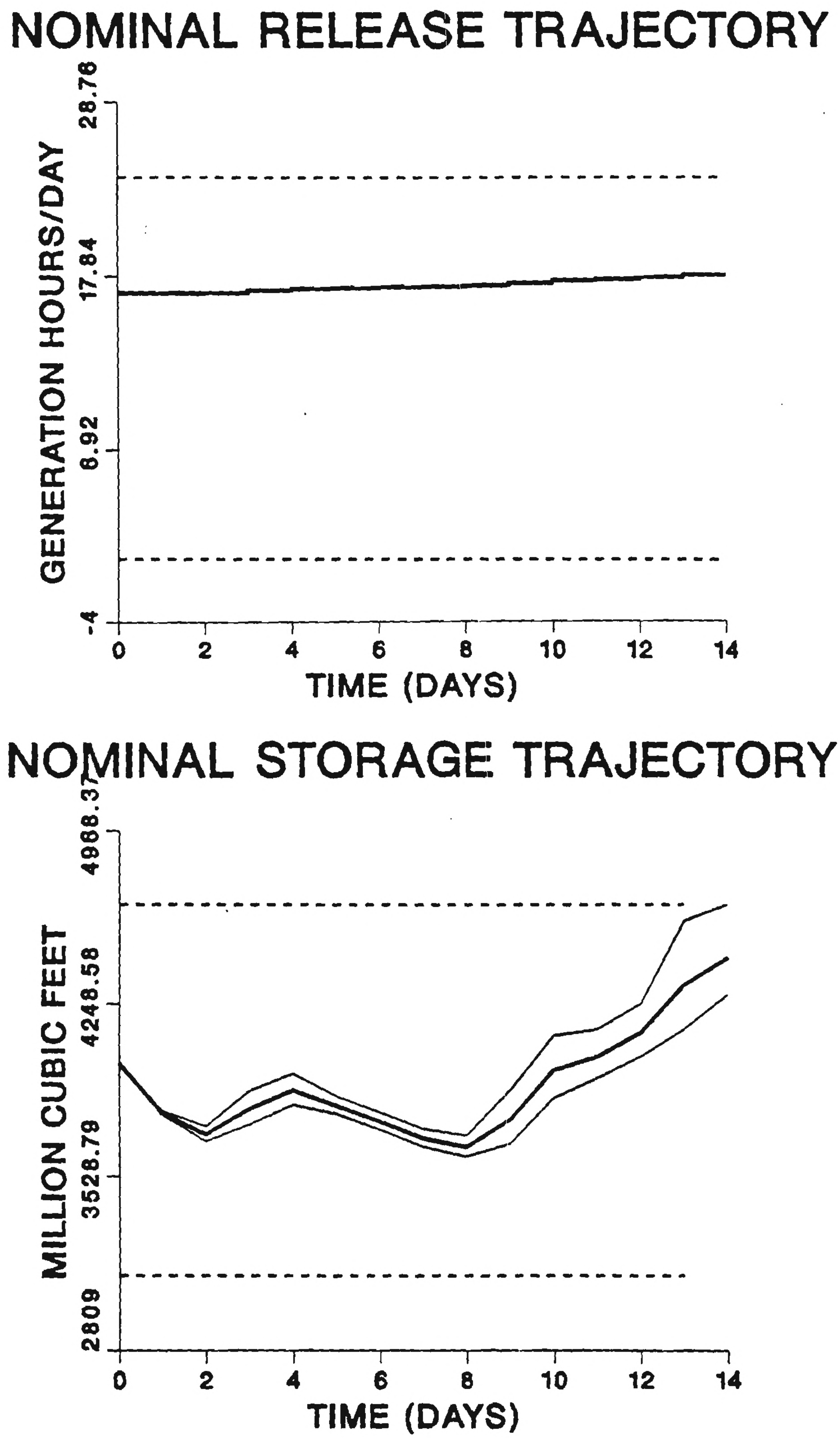


Figure 11: Optimal Sequences for Experiment #2

Table 7: Optimal Schedules for Experiment #2

OPTIMAL RELEASE AND POWER GENERATION SCHEDULES
STARTING DATE: 1/1/1988

PERIOD 1

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	603.88	636.09	599.93	644.44	661.59	702.09
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	37.08	76.65	27.41	37.77	39.47	45.75

GENERATION TIME: 16.81 HRS
FORECASTED END-OF-PERIOD STORAGE: 3793.43 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.55 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 364.18 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 2

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	611.52	643.97	607.43	652.72	670.09	711.63
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.81	76.10	27.21	37.50	39.19	45.42

GENERATION TIME: 16.78 HRS
FORECASTED END-OF-PERIOD STORAGE: 3704.96 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.04 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 367.54 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 3

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	615.01	647.56	610.86	656.51	673.98	716.00
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.69	75.85	27.12	37.38	39.06	45.27

GENERATION TIME: 16.82 HRS
FORECASTED END-OF-PERIOD STORAGE: 3811.63 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.65 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 369.06 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 4

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	610.82	643.24	606.75	651.96	669.31	710.75
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.84	76.15	27.23	37.53	39.21	45.45

GENERATION TIME: 16.96 HRS
FORECASTED END-OF-PERIOD STORAGE: 3889.82 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.10 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 367.24 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 5

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	607.87	640.20	603.85	648.76	666.02	707.06
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.94	76.36	27.31	37.63	39.32	45.58

GENERATION TIME: 17.08 HRS
FORECASTED END-OF-PERIOD STORAGE: 3825.22 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.73 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 365.95 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 6

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	610.30	642.71	606.24	651.40	668.73	710.10
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.85	76.18	27.24	37.55	39.23	45.47

GENERATION TIME: 17.13 HRS
FORECASTED END-OF-PERIOD STORAGE: 3758.21 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.35 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 367.01 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 7

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	612.89	645.38	608.78	654.21	671.62	713.35
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.76	76.00	27.18	37.46	39.14	45.36

GENERATION TIME: 17.18 HRS
FORECASTED END-OF-PERIOD STORAGE: 3687.85 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 524.94 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 368.14 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 8

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	615.70	648.27	611.54	657.26	674.75	716.87
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.67	75.80	27.11	37.36	39.04	45.25

GENERATION TIME: 17.23 HRS
FORECASTED END-OF-PERIOD STORAGE: 3652.06 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 524.73 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 369.36 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 9

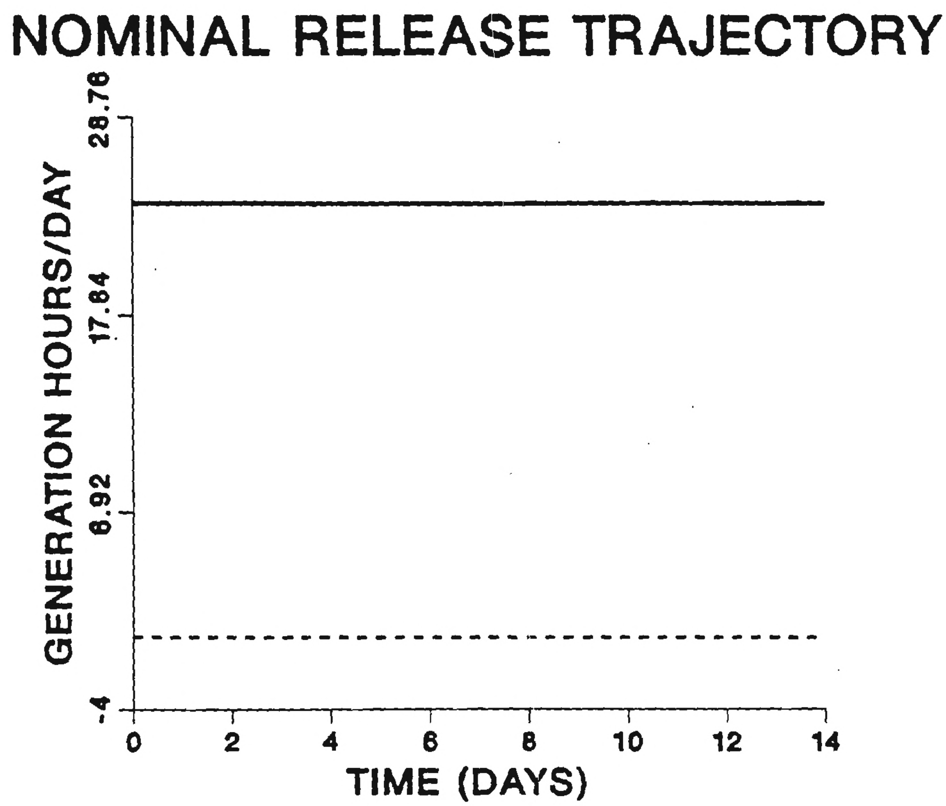
TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	617.16	649.78	612.98	658.85	676.38	718.71
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.62	75.70	27.07	37.31	38.98	45.18

GENERATION TIME: 17.29 HRS
FORECASTED END-OF-PERIOD STORAGE: 3764.79 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.39 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 369.99 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS

PERIOD 10

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	612.63	645.11	608.53	653.93	671.33	713.02
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	36.77	76.02	27.18	37.47	39.15	45.37

GENERATION TIME: 17.43 HRS
FORECASTED END-OF-PERIOD STORAGE: 3971.05 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 526.54 FT
MINIMUM GENERATION: 2000.00 KW FROM TURBINE #1 AT 368.03 CFS DISCHARGE
FLOOD GATE RELEASE: 0.00 CFS
FULL GATE OPERATION: 0.00 HRS



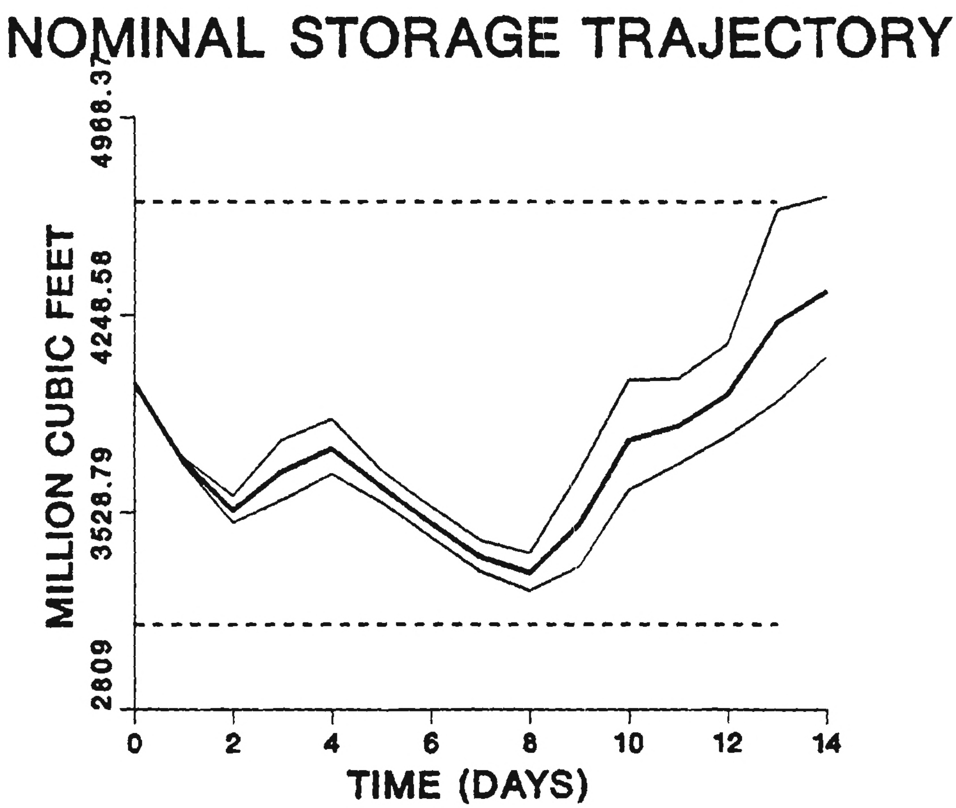


Figure 12: Optimal Sequences for Experiment #3

<u>Table 8:</u> Optimal Schedules for Experiment #3

OPTIMAL RELEASE AND POWER GENERATION SCHEDULES
STARTING DATE: 1/1/1988

PERIOD 1

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	609.41	641.79	605.36	650.43	667.74	708.99
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3712.45 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.08 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 1308.55 CFS
FULL GATE OPERATION: 3.48 HRS

PERIOD 2

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	621.05	653.78	616.79	663.08	680.72	723.61
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3535.20 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 524.02 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 1006.43 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 3

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	627.77	660.70	623.38	670.39	688.22	732.12
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3676.49 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 524.87 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 810.18 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 4

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	622.17	654.94	617.89	664.30	681.97	725.03
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3762.04 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.37 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 980.00 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 5

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	618.85	651.53	614.64	660.69	678.27	720.85
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3621.36 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 524.54 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 1062.48 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 6

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	624.07	656.89	619.76	666.36	684.09	727.43
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3487.83 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 523.73 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 930.37 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 7

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	629.12	662.09	624.71	671.86	689.74	733.84
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3362.82 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 522.95 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 767.97 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 8

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	634.16	667.27	629.65	677.36	695.38	740.27
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3304.07 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 522.57 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 619.40 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 9

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	636.58	669.76	632.02	680.00	698.09	743.37
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3482.08 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 523.70 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 552.95 CFS
FULL GATE OPERATION: 24.00 HRS

PERIOD 10

TURBINE #	1	2	3	4	5	6
DISCHARGE (CFS):	628.98	661.95	624.57	671.71	689.58	733.67
POWER (KW):	3200.00	3200.00	3200.00	3200.00	3400.00	3400.00
LEAKAGE (CFS)	0.00	0.00	0.00	0.00	0.00	0.00

GENERATION TIME: 24.00 HRS
FORECASTED END-OF-PERIOD STORAGE: 3790.04 MILLION CUBIC FEET
FORECASTED END-OF-PERIOD RESERVOIR LEVEL: 525.53 FT
MINIMUM GENERATION: COVERED BY ABOVE SCHEDULE
FLOOD GATE RELEASE: 772.23 CFS
FULL GATE OPERATION: 24.00 HRS

5.2 Simulation Experiments

Simulation experiments are intended to evaluate the system performance using the new control method. Among the issues to be investigated are the hydroelectric potential of Lloyd Shoals, the value of better inflow forecasts, the value of an additional (7th) turbine, and the hydroelectric losses due to turbine leakage.

5.2.1 Data Base

Simulation experiments may be conducted using historically observed or synthetically generated inflow sequences. In the case of the Lloyd Shoals Project, reservoir inflows can only be estimated through lake level and outflow discharge measurements. However, such records are not yet computerized and, therefore, prohibit the estimation of long historical inflow sequences. Thus, synthetic simulation experiments are only viable here. A brief description of the available data base follows.

(a) The rainfall data record includes daily values for the period from October 1980 through September 1981 from the five most relevant National Weather Service (NWS) stations located in Georgia. These stations are in Atlanta (WSO), Covington, Experiment, Jonesboro, and Monticello and are representative of the rainfall activity over the southern part of the Lloyd Shoals watershed. Unfortunately, the NWS station network does not adequately cover the upper basin. Mean areal precipitation estimates were obtained using the Thiessen estimation procedure (Figure 13).

(b) Evaporation data were compiled for October 1980 through September 1981 from two NWS stations located at Experiment and Athens. This study, however, primarily utilizes the Athens data (Figure 13) because the Experiment record is incomplete.

(c) The streamflow record includes daily values for the 1980-1981 water year from USGS station 02210500 on the Ocmulgee river near the City of Jackson. This station is approximately one mile downstream from Lloyd Shoals Dam, and its readings are considered representative of the total reservoir outflow.

(d) Daily reservoir elevations for the 1980-1981 water year were compiled from Georgia Power records. These values were used with the streamflow record to generate the sequence of net reservoir inflows to the Lloyd Shoals project. This analysis was performed using water balance considerations and the reservoir elevation versus storage relationship, derived from regression analysis of surveying data. A plot of the estimated net reservoir inflow values appears on Figure 13.

(e) The geomorphologic characteristics of the Lloyd Shoals watershed were compiled from USGS 1:100,000 scale maps. The watershed was divided into five subbasins drained by the Alcovy River, Yellow River, South River, Walnut Creek, and Tussahaw Creek. In each subbasin, each stream was delineated and ranked according to the Strahler ranking system. Then the following characteristics were measured and computerized: stream order, order of receptor stream, length, and slope. This information was processed for a total of 664 streams.

Based on the above geomorphoclimatic data, a physically-based rainfall-runoff model was calibrated and subsequently used to generate thirty years of daily net reservoir inflows. This model (Georgakakos and Kabouris, 1989) utilizes the concept of geomorphologic instantaneous unit hydrographs and requires a series of rainfall inputs. These inputs were generated using a

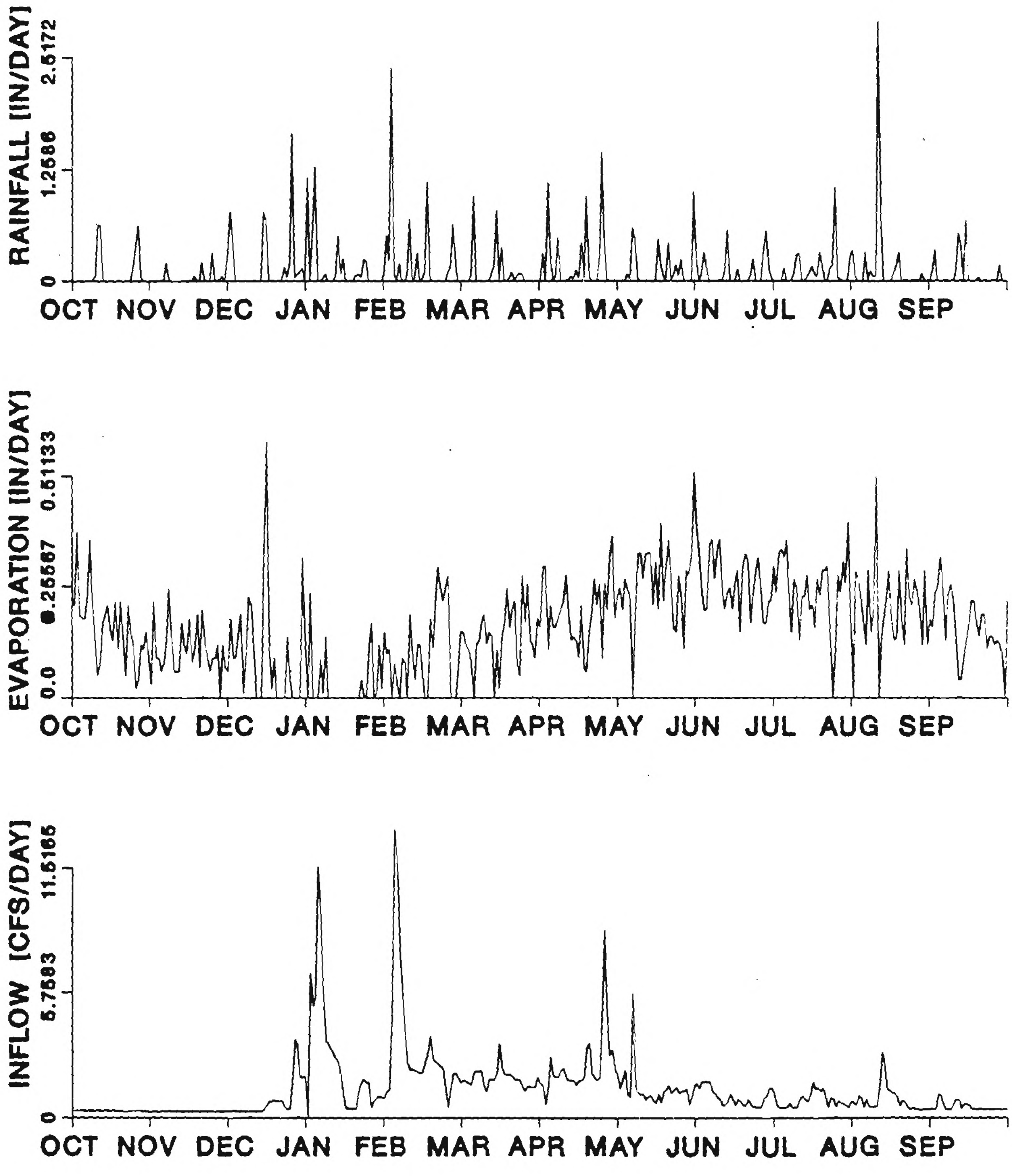


Figure 13: Rainfall, Evaporation, and Net Reservoir Inflows for Lloyd Shoals

(1980 - 1981 Water Year)

statistical rainfall model, which was developed as follows:

The daily rainfall data in (a) were grouped into the following two sets depending on season: one included the rainfall events over the rainy season (December through June) and the other included the events over the dry season (July through November). The assumption was made that the events in each group follow a Poisson process (Benjamin and Cornell, 1970, Eagleson, 1978) with exponentially-distributed storm durations and interstorm periods and gamma-distributed storm depths. More specifically, it was assumed that t_s (storm duration), t_i (interstorm period), and d (rainfall depth) have the following probability distributions:

$$f_{T_s}(t_s) = \lambda_s \, e^{-\lambda_s t_s} , \qquad (37)$$

$$f_{T_i}(t_i) = \lambda_i \, e^{-\lambda_i t_i} , \text{ and} \qquad (38)$$

$$f_D(d) = \frac{\lambda_d \, (\lambda_d \, d)^{\kappa_d - 1} \, e^{-\lambda_d d}}{\Gamma(\kappa_d)} , \qquad (39)$$

where $f_X(x)$ represents the probability density of the random variable X evaluated at x; λ_s, λ_i, λ_d, κ_d are calibration parameters; and $\Gamma(\kappa_d)$ is the incomplete gamma function:

$$\Gamma(\kappa_d) = \int_0^\infty e^{-x} \, x^{\kappa_d - 1} \, dx . \qquad (40)$$

The previous parameters were calibrated for each period and were set equal to the following estimated values:

Table 9: Parameters of the Rainfall Model

Parameter	Rainy Season (December - June)	Dry Season (July - November)
λ_s [days^{-1}]	0.2605	0.3200
λ_i [days^{-1}]	0.3263	0.3243
λ_d [inches^{-1}]	1.4045	0.7324
κ_d	1.4841	0.4480

The rational behind this simple rainfall generation model is to preserve the statistical nature of the basic storm characteristics (duration, interstorm period, and storm depth) for both the rainy and the dry seasons. A synthetic storm sequence can be developed by generating independent random values for the triplet $[t_i, d, t_r]$ that defines each storm event. In this study, the generated synthetic storm sequence was thirty years long. This sequence constituted the rainfall input for the previously mentioned rainfall-runoff model which was then used to generate a 30-year long sequence of net reservoir inflows. These inflows are the basis of the simulation process presented in the next section.

5.2.2 The Simulation Process

The simulation process (Figure 14) seeks to imitate the operation of the Lloyd Shoals reservoir under the guidance of the control model previously discussed. At the beginning of each day, an inflow forecasting model is first invoked to provide inflow forecasts over the control horizon. These forecasts become available to the control model which finds optimal power generation and discharge schedules according to the methodology presented in

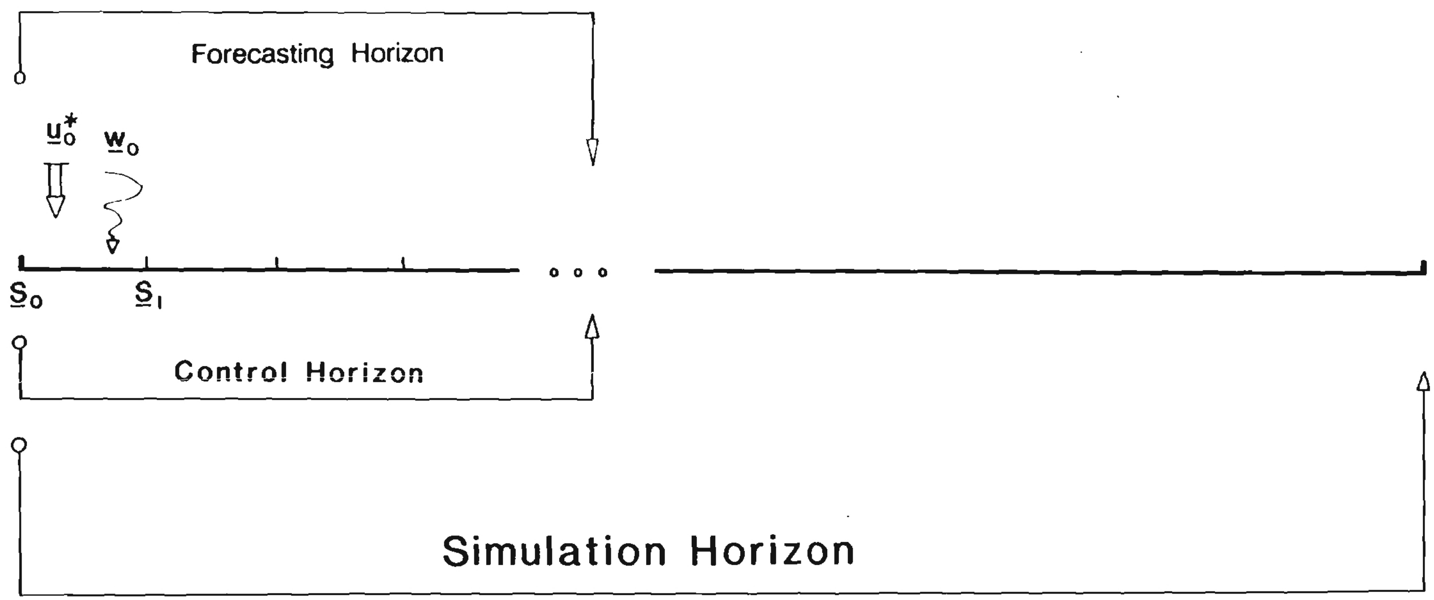


Figure 14: The Simulation Process

Section 4. The schedules for the first day are implemented, the actual inflow values are generated, and the system outputs (end–of–day reservoir storage, actual turbine release, leakage rates, flood gate and flash board releases, peak and off-peak power generation) are computed and recorded. This process is repeated at the beginning of each day for the duration of the simulation horizon (30 years).

The lack of sufficient hydrologic data prohibits the valid calibration of an inflow forecasting model. Instead, this model operation is herein simulated as follows: Let the daily, apriori, inflow means and variances be denoted as $\mu(k)$ and $\sigma^2(k)$ respectively. These statistics were obtained from the generated net reservoir inflow series. Then, at the beginning of each day, the forecasts (mean, $m(k)$, and forecast error variance, $s^2(k)$) are computed by the following process:

1. Generate N independent standard normal variables $\omega(k)$, $k=1,\ldots,N$.

2. Transform the previous variables into normal variables with mean $q(k)$ and variance $[0.05\, q(k)\, (1-\rho)]^2$, where $q(k)$ is the net reservoir inflow for day k and ρ is a forecast parameter in the range $[0,1]$.

$$w(k) = \omega(k)\ 0.05\ q(k)\ [1 - \rho] + q(k),\ k=1,\ldots,N. \tag{41}$$

3. Forecast the upcoming inflows from

$$m(k) = \mu(k) + [w(k) - \mu(k)]\ \rho^k,\ k=1,\ldots,N. \tag{42}$$

4. Compute the associated forecast error variance from

$$s^2(k) = \sigma^2(k)\ [1 - (\rho^2)^k],\ k=1,\ldots,N. \tag{43}$$

This procedure has the following characteristics:

(i) For a given ρ, the forecast statistics tend to the apriori means $\mu(k)$ and the associated variances $\sigma^2(k)$ with time. This becomes evident from Equations (42) and (43) as ρ^k tends to 0 and $[1 - (\rho^2)^k]$ tends to 1.

(ii) As ρ tends to 1, the forecasting accuracy improves, since w(k) tends to q(k), m(k) tends to w(k), and $s^2(k)$ tends to zero. For ρ=1, this model generates perfect forecasts.

(iii) As ρ tends to 0, the forecasting accuracy deteriorates, since m(k) tends to $\mu(k)$ and $s^2(k)$ tends to $\sigma^2(k)$. For ρ=0, the previous scheme simply generates the apriori statistics.

(iv) For realism, the forecasts are assumed to follow lognormal probability distributions.

Thus, at the beginning of each day and depending on the value of ρ, the previous model generates inflow forecasts for the upcoming days of the control horizon. Then, the control model determines the optimal power generation and release schedules, and the system response is simulated. For accuracy, this simulation is performed on hourly intervals and is based on the following assumptions:

(a) If the optimal generation time schedule, $t^*(k)$, is less than or equal to 16 hours, power generation is assumed to begin at 8:00 am; otherwise, power generation begins at $[24 - t^*(k)]$ am and ends at 12:00 midnight.

(b) If the optimal decision calls for flood gate releases, flood gate operation begins at 12:00 midnight.

(c) If at any time within the day, the reservoir level exceeds the threshold of 530 feet, the flash boards trip and cannot be reset until the water level has receded below elevation 528 feet. To maximize energy generation during this time, the flood gate is completely shut, and all turbines operate at maximum power. The spillway outflow is determined from

$$Q_s = 0.2547755 \times 10^7 - 0.14145744 \times 10^5 H + 0.2615793 \times 10^2 H^2 - 0.16108972 \times 10^{-1} H^3, \quad (41)$$

where H is the reservoir elevation in feet and Q_S is the spillway outflow in thousands of cfs.

(d) The hourly net reservoir inflow is equal to q(k)/24, where q(k) is the daily synthetic inflow value generated as explained in Section 5.2.1.

(e) For each hourly interval, the simulation routine computes the following quantities: end-of-the-hour reservoir storage, turbine release and leakage, spillway and flood gate outflows, and peak and off-peak power generation. At the end of each day, the corresponding daily values are determined by summation.

5.2.3 Five Simulation Experiments

The simulation experiments differ in the values of the forecast parameter ρ, the tolerance levels γ, the number of the operational turbines, and the turbine leakage rate. Some common features are that the initial storage is set equal to 4,247 million cubic feet and that the control horizon is 14 days. More specifically, the characteristics of each experiment are summarized below:

Table 10: Characteristics of the Simulation Experiments

	Simulation Experiment I	II	III	IV	V
ρ	0.01	0.01	0.01	0.90	0.90
γ	50%	2.5%	2.5%	2.5%	2.5%
Turbines	6	6	7	6	6
Leakage	ℓ^{ref}	ℓ^{ref}	ℓ^{ref}	ℓ^{ref}	0.01 ℓ^{ref}

In the previous table, the 50% value for γ indicates that the corresponding control model is only concerned with maintaining the mean storage sequence within the storage bounds (deterministic optimization). When γ=2.5%, the storage constraints are satisfied 2.5% of the time (stochastic optimization). The turbines of Model V are assumed to have parameters ℓ^{ref} which are ten times lower than the values reported in Table 1.

As an example of the results generated by each simulation experiment, Table 11 is an excerpt for a selected 16–day period from experiments I and IV. The table reports the date, end-of-period storage; flood gate, flash board, and turbine releases; leakage and inflow rates; and peak and off-peak energy generation. This table is also indicative of how each control model performs differently. It is notable that control model I exhibits a reactive type of response and cannot help frequently tripping the flash-boards. Model IV, on the other hand, having the benefit of good forecasts, is able to anticipate high flows, promptly adjust turbine releases, and avoid spillage. As a result between the two models, Model IV produces considerably more peak and off–peak energy.

Statistical analysis was employed to evaluate the performance of each control model over the 30–year simulation period. Table 11 reports the results of this analysis on a yearly basis; the monthly statistics are presented in Tables 12 through 16. Each table includes the mean and 5% and 95% percentiles of the reservoir storage, flood gate and flash board releases, turbine release and leakage, inflow, peak and off–peak energy generation. Several comments are now in order.

1. Models I and II differ only in the value of the tolerance level γ (50% and 2.5% respectively). Smaller values of γ maintain lower reservoir storages, lessen the flash board mandatory releases, and, consequently, generate more peak and off–peak energy. Thus on the average, Model II

generates approximately 1,000 MWH more peak and 2,500 MWH more off-peak energy per year and experiences almost half the spillway losses of Model I.

2. Models II and III have the same forecasting and tolerance levels but differ in the number of turbines. Model III is assumed to have an additional (7th) turbine which is identical to Turbine #1 but leaks at a rate ten times lower. As expected, the addition of the 7th turbine causes an average increase in peak energy generation by about 830 MWH per year and a corresponding decrease in off-peak energy generation. This improvement is more substantial in "wet" seasons where peak annual generation may improve by 1,500 MWH (95% percentile).

3. Models II and IV differ by the accuracy of the forecasting scheme. More accurate streamflow forecasting allows the reservoir to maintain higher storage levels without compromising constraint reliability. As a result, flash board outflow was drastically reduced from 2,631 x10^6 ft^3 to 261 x10^6 ft^3, while average annual peak and off-peak energy generation rose by 1,300 and 2,800 MWH respectively. Based on average system production costs shown in Figure 1, these improvements represent an approximate average gain of $100,000 per year. The actual gains during "wet" years are considerably higher, while the value of forecasting declines during "dry" years.

4. A comparison of the results for Models I and IV reveals the value of stochastic control methods with accurate streamflow forecasting. The improvements amount to a yearly average of 1,300 MWH peak and 5,600 MWH off-peak energy production and an economic gain of approximately $135,000. These results demonstrate that fully stochastic optimal control methods in connection with streamflow forecasting can substantially improve hydropower revenues.

5. Models IV and V differ by the turbine leakage rates. The turbines of Model V leak at a rate ten times lower than Model IV. The results indicate that the average peak and off-peak energy generation of Model V improved by 5,250 and 2,000 MWH respectively over those of Model IV. Based on Figure 1, these improvements amount to about $300,000 to $350,000 per year. Similar improvements are also realized in the 95% reliable energy output. In this regard, peak energy improved by 6,000 MWH, and off-peak energy by 2,600 MWH.

Figures 15 through 44 include plots of the simulation data and their daily and monthly frequencies. For clarity, the simulation data are plotted for the first 5,000 days. In the frequency plots, the thicker lines delineate the mean trajectories, while the thinner ones, the 95% probability limits. The frequency plots and tables can be used to determine the 95% reliable peak and off-peak energy outputs.

The following section summarizes the results of this work and offers further recommendations.

Table 11: Annual Statistics for the Simulation Experiments

Expmnt.	Storage 10^6 cf	Flood Gate Release 10^6 cf	Flash Board Release 10^6 cf	Turbine Release 10^6 cf	Leakage 10^6 cf	Inflow 10^6 cf	Peak Power MWH	Off-Pk. Power MWH
I								
5%	2,779	0	188	12,641	5,136	19,619	13,138	4,163
Mean	3,635	0	5,150	26,751	5,737	37,446	28,998	9,320
95%	3,906	0	11,969	37,367	6,411	51,921	40,587	13,643
II								
5%	2,515	0	0	12,947	4,857	19,619	13,724	4,231
Mean	3,504	260	2,631	29,162	5,528	37,446	29,791	12,091
95%	3,876	1,026	6,099	41,394	6,213	51,921	41,508	19,479
III								
5%	2,537	0	0	13,145	5,243	19,619	13,997	4,095
Mean	3,461	76	2,492	29,209	5,799	37,446	30,621	11,333
95%	3,841	348	5,772	41,187	6,365	51,921	43,009	18,258
IV								
5%	2,947	0	0	13,382	4,705	19,619	12,651	5,415
Mean	3,810	455	262	31,208	5,556	37,446	30,277	14,926
95%	4,063	1,918	1,891	45,331	6,551	51,921	43,061	24,592
V								
5%	3,575	0	0	19,251	448	19,619	18,612	8,058
Mean	3,896	470	261	36,216	533	37,446	35,515	16,888
95%	4,108	2,044	1,891	49,483	634	51,921	46,824	26,240

Table 12: Monthly Simulation Statistics for Experiment I

Month		Storage 10^6 cf	Fl. Gt. Release 10^6 cf	Fl. Bd. Release 10^6 cf	Turbine Release 10^6 cf	Leakage 10^6 cf	Inflow 10^6 cf	Peak Power MWH	Off-Pk. Power MWH
	5 %	3059.7	0.0	0.0	188.3	271.2	753.9	167.2	87.3
J	Mean	3581.7	0.0	488.2	2589.2	470.5	3465.0	2792.0	905.8
	95 %	4360.7	0.0	3635.3	5979.5	584.2	9897.0	5703.4	2899.5
	5 %	3152.6	0.0	0.0	433.9	290.4	845.6	334.3	221.0
F	Mean	3625.7	0.0	392.2	2524.9	419.6	3406.7	2824.0	781.9
	95 %	4396.6	0.0	2136.6	4785.2	541.5	7186.6	5270.4	1782.5
	5 %	2976.5	0.0	0.0	9.9	333.3	259.0	7.7	2.8
M	Mean	3539.0	0.0	275.9	2617.2	469.0	3212.1	2921.7	806.6
	95 %	4195.6	0.0	1943.3	4956.9	585.2	7791.3	5494.0	1985.7
	5 %	2708.8	0.0	0.0	0.0	264.6	107.0	0.0	0.0
A	Mean	3531.7	0.0	518.7	2551.2	445.8	3537.2	2810.5	828.2
	95 %	4387.7	0.0	3226.1	5731.0	567.2	9493.6	5795.9	2465.1
	5 %	2644.9	0.0	0.0	0.0	265.9	134.8	0.0	0.0
M	Mean	3565.1	0.0	700.9	2413.3	473.7	3880.5	2581.2	858.5
	95 %	4406.5	0.0	5323.4	6127.6	585.3	11675.7	5449.3	3332.6
	5 %	2185.3	0.0	0.0	0.0	353.5	51.1	0.0	0.0
J	Mean	3872.6	0.0	684.3	1458.8	515.7	2689.5	1353.7	764.9
	95 %	4422.2	0.0	4517.4	4529.2	588.6	9270.2	3618.1	2938.2
	5 %	2048.2	0.0	0.0	0.0	405.8	161.9	0.0	0.0
J	Mean	3989.0	0.0	499.3	1603.2	531.7	2784.5	1555.9	779.9
	95 %	4392.7	0.0	3519.4	3982.9	609.2	7815.3	3546.8	2373.7
	5 %	1664.4	0.0	0.0	0.0	359.5	83.8	0.0	0.0
A	Mean	4068.8	0.0	572.8	1995.9	510.4	2979.5	2073.7	835.0
	95 %	4417.3	0.0	2968.9	4676.8	607.2	7878.2	4499.9	2450.7
	5 %	1269.5	0.0	0.0	0.0	305.9	213.3	0.0	0.0
S	Mean	3495.6	0.0	273.5	2691.5	438.2	2705.2	3123.9	724.0
	95 %	4332.5	0.0	2058.9	5094.9	542.4	6895.1	5575.1	1784.0
	5 %	1612.0	0.0	0.0	0.0	301.9	284.2	0.0	0.0
O	Mean	3319.4	0.0	277.8	1936.5	496.7	2720.9	2093.2	664.1
	95 %	4249.9	0.0	2133.5	5471.1	581.9	7984.7	5533.8	2311.6
	5 %	2965.3	0.0	0.0	109.3	341.4	321.7	75.1	39.6
N	Mean	3443.3	0.0	123.9	2022.3	482.7	2890.7	2269.8	605.7
	95 %	4291.9	0.0	1142.2	4550.0	565.1	6456.3	5439.9	1143.1
	5 %	2961.9	0.0	0.0	0.0	315.2	562.9	0.0	0.0
D	Mean	3586.5	0.0	342.8	2346.8	483.2	3174.0	2598.0	765.0
	95 %	4352.8	0.0	2385.7	5262.1	585.6	7417.7	5739.7	2047.0

Table 13: Monthly Simulation Statistics for Experiment II

Month		Storage 10^6 cf	Fl. Gt. Release 10^6 cf	Fl. Bd. Release 10^6 cf	Turbine Release 10^6 cf	Leakage 10^6 cf	Inflow 10^6 cf	Peak Power MWH	Off-Pk. Power MWH
	5 %	3153.2	0.0	0.0	295.9	206.4	753.9	201.9	218.7
J	Mean	3740.5	29.8	284.9	2707.7	469.5	3465.0	2614.8	1305.9
	95 %	4470.3	222.6	2371.7	6973.2	588.4	9897.0	5474.5	4608.0
	5 %	3244.3	0.0	0.0	433.0	216.8	845.6	287.5	282.4
F	Mean	3812.2	12.1	215.1	2670.8	415.9	3406.7	2691.4	1175.4
	95 %	4458.2	95.7	1142.5	5977.8	546.1	7186.6	5145.3	3556.0
	5 %	3072.1	0.0	0.0	0.0	282.0	259.0	0.0	0.0
M	Mean	3720.4	11.3	203.5	2702.2	470.7	3212.1	2807.9	1094.4
	95 %	4331.7	127.0	1286.0	5747.8	588.2	7791.3	5345.3	2999.3
	5 %	2803.4	0.0	0.0	0.0	126.7	107.0	0.0	0.0
A	Mean	3658.6	70.2	171.3	2873.2	430.0	3537.2	2741.4	1401.0
	95 %	4451.5	599.0	1058.5	7831.5	570.0	9493.6	5833.6	5562.6
	5 %	2750.7	0.0	0.0	0.0	131.4	134.8	0.0	0.0
M	Mean	3629.9	72.2	262.0	2994.8	440.8	3880.5	2970.0	1330.5
	95 %	4436.3	475.3	3110.9	8140.6	588.3	11675.7	6085.5	5618.1
	5 %	1731.5	0.0	0.0	750.8	256.1	51.1	813.3	119.0
J	Mean	3433.5	10.1	311.6	2365.4	443.9	2689.5	2476.5	877.5
	95 %	4453.7	102.4	3121.3	5853.5	509.5	9270.2	5010.2	3560.4
	5 %	1185.4	0.0	0.0	0.0	327.6	161.9	0.0	0.0
J	Mean	3060.0	4.0	166.4	2277.0	460.0	2784.5	2452.0	733.4
	95 %	4315.9	56.9	1554.6	5004.1	531.2	7815.3	4643.6	2941.8
	5 %	810.5	0.0	0.0	0.0	312.7	83.8	0.0	0.0
A	Mean	3162.2	11.8	281.4	2260.6	461.1	2979.5	2420.9	759.8
	95 %	4431.4	159.7	2201.7	5284.6	528.9	7878.2	4842.4	2805.0
	5 %	484.8	0.0	0.0	0.0	265.8	213.3	0.0	0.0
S	Mean	3279.2	9.9	96.3	2081.2	464.4	2705.2	2174.3	837.7
	95 %	4372.2	145.8	1001.2	5712.6	560.4	6895.1	5362.6	2926.6
	5 %	917.9	0.0	0.0	0.0	252.9	284.2	0.0	0.0
O	Mean	3280.1	16.5	173.4	1984.8	489.9	2720.9	1971.3	894.4
	95 %	4393.8	169.8	1516.0	6225.7	586.2	7984.7	5633.1	3378.7
	5 %	2100.0	0.0	0.0	0.0	321.0	321.7	0.0	0.0
N	Mean	3521.9	7.9	108.7	1899.8	491.2	2890.7	2009.6	738.2
	95 %	4338.9	89.3	992.2	4877.5	568.3	6456.3	5181.7	2199.8
	5 %	2864.1	0.0	0.0	0.0	340.7	562.9	0.0	0.0
D	Mean	3749.4	4.1	356.4	2344.9	490.6	3174.0	2460.9	942.7
	95 %	4375.1	49.3	2544.0	4934.6	589.2	7417.7	5202.7	2172.1

Table 14: Monthly Simulation Statistics for Experiment III

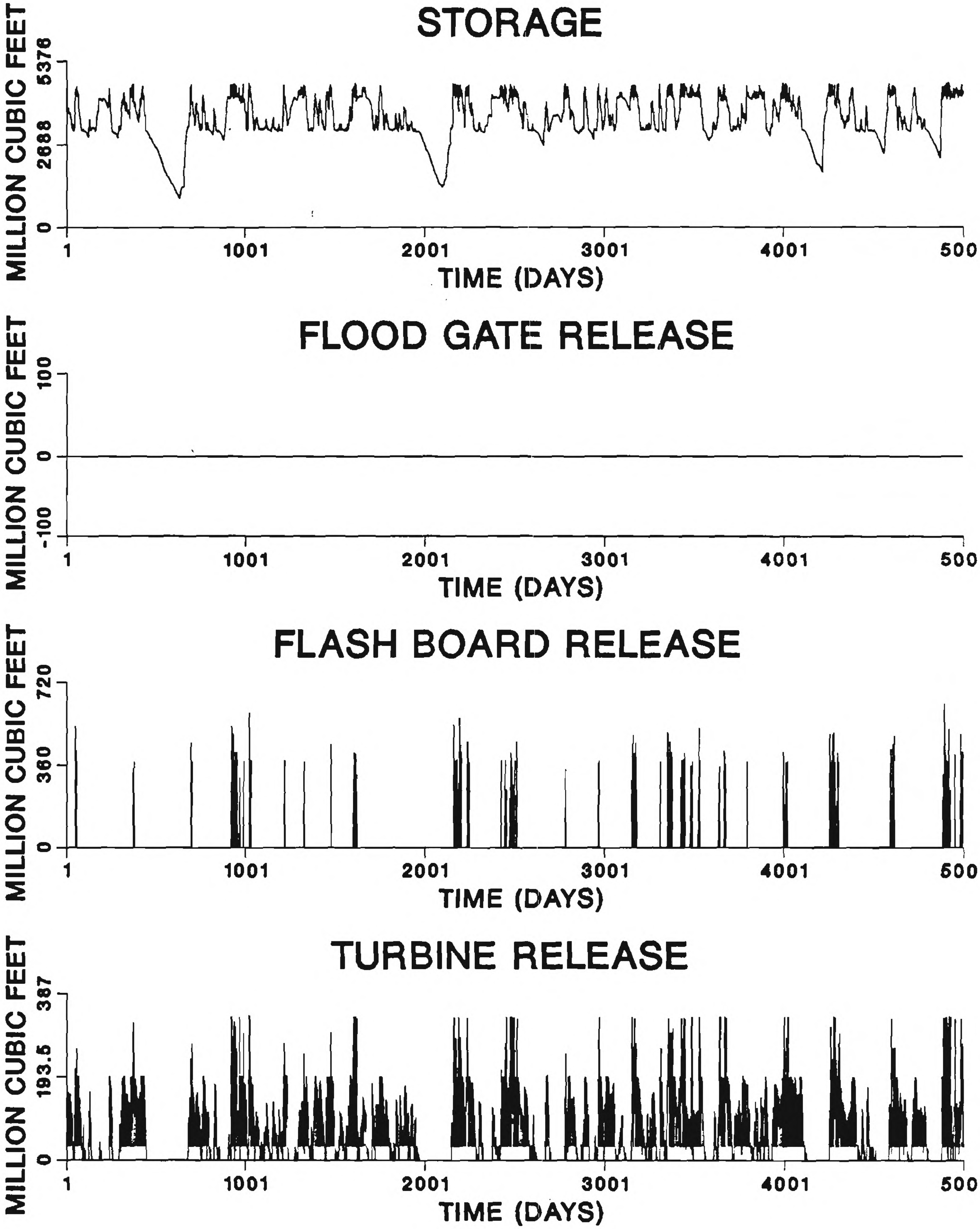
Month		Storage 10^6 cf	Fl. Gt. Release 10^6 cf	Fl. Bd. Release 10^6 cf	Turbine Release 10^6 cf	Leakage 10^6 cf	Inflow 10^6 cf	Peak Power MWH	Off-Pk. Power MWH
	5 %	3116.5	0.0	0.0	286.2	271.9	753.9	189.6	183.5
J	Mean	3726.6	7.5	289.9	2692.2	496.1	3465.0	2645.1	1258.3
	95 %	4417.2	84.9	2373.0	6948.6	598.4	9897.0	5810.4	4452.8
	5 %	3244.9	0.0	0.0	424.0	269.9	845.6	281.1	282.4
F	Mean	3802.3	1.5	229.9	2655.1	442.2	3406.7	2766.3	1084.4
	95 %	4452.0	20.6	1304.6	6007.0	556.7	7186.6	5581.1	3169.8
	5 %	3064.8	0.0	0.0	0.0	329.7	259.0	0.0	0.0
M	Mean	3709.8	2.2	189.7	2688.0	497.2	3212.1	2849.4	1040.7
	95 %	4319.3	32.2	1192.4	5858.2	597.9	7791.3	5793.4	2732.7
	5 %	2795.5	0.0	0.0	0.0	187.7	107.0	0.0	0.0
A	Mean	3643.7	17.4	173.0	2919.2	455.2	3537.2	2927.2	1288.8
	95 %	4424.5	154.3	1153.7	8070.4	579.1	9493.6	6494.5	5184.7
	5 %	2736.8	0.0	0.0	0.0	180.3	134.8	0.0	0.0
M	Mean	3604.7	27.1	236.3	3025.1	467.8	3880.5	3090.6	1260.0
	95 %	4418.9	241.8	2800.2	8609.0	597.6	11675.7	6949.5	5471.4
	5 %	1719.1	0.0	0.0	697.0	310.0	51.1	782.5	86.4
J	Mean	3359.8	.6	325.1	2439.3	461.7	2689.5	2637.5	814.8
	95 %	4431.5	8.8	3323.0	5862.4	516.9	9270.2	5329.9	3124.7
	5 %	1264.7	0.0	0.0	0.0	375.2	161.9	0.0	0.0
J	Mean	2913.1	0.0	144.6	2300.2	476.5	2784.5	2545.5	643.1
	95 %	4298.0	0.0	1435.3	4953.5	538.8	7815.3	4717.7	2719.7
	5 %	880.9	0.0	0.0	0.0	343.4	83.8	0.0	0.0
A	Mean	3026.6	2.9	232.6	2314.4	477.0	2979.5	2529.1	714.5
	95 %	4402.3	39.9	1774.0	5554.8	534.6	7878.2	5293.5	2746.8
	5 %	541.6	0.0	0.0	0.0	309.8	213.3	0.0	0.0
S	Mean	3227.2	1.8	92.4	1943.6	490.5	2705.2	2089.6	730.4
	95 %	4364.8	24.5	949.6	5870.2	576.1	6895.1	5782.9	2738.7
	5 %	960.7	0.0	0.0	0.0	293.3	284.2	0.0	0.0
O	Mean	3266.2	9.3	148.2	1992.1	510.3	2720.9	2003.5	879.7
	95 %	4384.6	117.1	1226.4	6496.8	594.9	7984.7	6023.1	3469.1
	5 %	2047.1	0.0	0.0	0.0	370.8	321.7	0.0	0.0
N	Mean	3510.3	4.9	105.2	1879.0	511.6	2890.7	2023.9	699.6
	95 %	4294.1	72.8	1061.7	4772.1	577.5	6456.3	5311.4	1844.6
	5 %	2798.7	0.0	0.0	0.0	369.0	562.9	0.0	0.0
D	Mean	3738.3	.6	324.5	2360.4	512.7	3174.0	2513.3	919.0
	95 %	4370.5	9.1	2172.4	5218.2	599.1	7417.7	5430.4	2230.3

Table 15: Monthly Simulation Statistics for Experiment IV

Month		Storage 10^6 cf	Fl. Gt. Release 10^6 cf	Fl. Bd. Release 10^6 cf	Turbine Release 10^6 cf	Leakage 10^6 cf	Inflow 10^6 cf	Peak Power MWH	Off-Pk. Power MWH
	5 %	3252.8	0.0	0.0	340.9	132.9	753.9	219.5	245.4
J	Mean	3784.7	64.2	53.9	2905.3	460.9	3465.0	2753.2	1444.4
	95 %	4460.2	770.9	765.7	8082.1	590.0	9897.0	5875.7	5844.0
	5 %	3301.8	0.0	0.0	454.1	162.3	845.6	273.4	310.1
F	Mean	3842.4	16.0	0.0	2903.6	405.9	3406.7	2818.9	1374.7
	95 %	4529.3	180.5	0.0	6792.7	546.5	7186.6	5362.8	4607.2
	5 %	3151.5	0.0	0.0	8.0	226.4	259.0	5.2	2.8
M	Mean	3772.6	20.0	0.0	2885.3	463.6	3212.1	2937.8	1221.5
	95 %	4295.7	274.3	0.0	6670.6	589.2	7791.3	5711.5	3983.4
	5 %	2884.4	0.0	0.0	0.0	66.6	107.0	0.0	0.0
A	Mean	3711.0	45.6	0.0	3060.7	424.6	3537.2	2841.2	1578.5
	95 %	4440.2	597.7	0.0	8738.7	571.1	9493.6	5983.5	6684.6
	5 %	2798.7	0.0	0.0	0.0	64.1	134.8	0.0	0.0
M	Mean	3712.1	91.5	89.7	3100.8	438.5	3880.5	2985.7	1484.6
	95 %	4470.6	1195.5	1211.3	9133.7	589.5	11675.7	6231.4	6955.6
	5 %	2335.9	0.0	0.0	0.0	133.9	51.1	0.0	0.0
J	Mean	3969.0	66.9	31.8	1956.4	484.8	2689.5	1738.8	1131.4
	95 %	4525.4	829.6	429.1	7729.1	592.3	9270.2	5634.7	5572.8
	5 %	2189.5	0.0	0.0	0.0	221.6	161.9	0.0	0.0
J	Mean	4087.2	44.2	23.4	2084.5	502.9	2784.5	1913.6	1148.5
	95 %	4478.4	455.8	315.7	6698.4	612.5	7815.3	4875.8	4996.9
	5 %	1791.1	0.0	0.0	0.0	181.8	83.8	0.0	0.0
A	Mean	4164.3	37.2	0.0	2520.6	479.1	2979.5	2372.7	1328.4
	95 %	4509.4	462.3	0.0	7301.1	609.4	7878.2	5636.3	4979.3
	5 %	1389.7	0.0	0.0	0.0	200.4	213.3	0.0	0.0
S	Mean	3747.2	19.0	0.0	2800.9	443.7	2705.2	2893.7	1170.4
	95 %	4440.6	278.4	0.0	6710.3	563.7	6895.1	5731.8	4237.4
	5 %	1725.7	0.0	0.0	0.0	192.2	284.2	0.0	0.0
O	Mean	3525.1	17.5	12.7	2183.9	493.8	2720.9	2048.4	1098.5
	95 %	4408.3	185.7	171.6	7212.2	590.3	7984.7	5773.3	4663.5
	5 %	3120.1	0.0	0.0	118.9	279.9	321.7	74.7	64.4
N	Mean	3638.6	2.8	0.0	2177.9	482.9	2890.7	2274.4	854.4
	95 %	4448.7	37.4	0.0	5512.5	570.9	6456.3	5359.6	2720.9
	5 %	3140.1	0.0	0.0	10.2	238.8	562.9	8.2	2.8
D	Mean	3763.5	30.1	50.3	2627.7	475.1	3174.0	2698.2	1090.6
	95 %	4417.0	383.9	547.0	6499.7	590.4	7417.7	5969.5	3696.1

Table 16: Monthly Simulation Statistics for Experiment V

Month		Storage 10⁶ cf	Fl. Gt. Release 10⁶ cf	Fl. Bd. Release 10⁶ cf	Turbine Release 10⁶ cf	Leakage 10⁶ cf	Inflow 10⁶ cf	Peak Power MWH	Off-Pk. Power MWH
	5 %	3309.2	0.0	0.0	825.4	12.5	753.9	676.3	483.4
J	Mean	3807.4	63.6	53.9	3342.9	43.8	3465.0	3217.8	1608.9
	95 %	4481.8	771.4	765.7	8208.9	58.3	9897.0	5935.5	5971.5
	5 %	3294.2	0.0	0.0	961.5	15.7	845.6	724.4	565.6
F	Mean	3851.6	15.4	0.0	3276.2	38.6	3406.7	3224.6	1511.2
	95 %	4535.1	180.8	0.0	6912.3	53.4	7186.6	5426.8	4767.8
	5 %	3244.3	0.0	0.0	288.3	21.4	259.0	164.7	168.2
M	Mean	3792.5	20.7	0.0	3277.3	44.2	3212.1	3364.6	1351.7
	95 %	4320.3	254.4	0.0	6874.2	58.9	7791.3	5731.2	4217.2
	5 %	3227.5	0.0	0.0	98.0	6.1	107.0	42.1	75.4
A	Mean	3761.0	48.1	0.0	3409.7	40.7	3537.2	3202.0	1708.0
	95 %	4452.6	614.3	0.0	8816.5	57.0	9493.6	5996.8	6790.3
	5 %	3221.3	0.0	0.0	96.9	5.9	134.8	43.4	57.3
M	Mean	3792.8	92.1	89.7	3442.0	42.2	3880.5	3309.3	1640.2
	95 %	4474.9	1201.4	1211.3	9211.2	59.0	11675.7	6257.3	7043.6
	5 %	3215.0	0.0	0.0	0.0	12.6	51.1	0.0	0.0
J	Mean	4143.1	66.9	31.8	2309.6	47.3	2689.5	2094.6	1300.7
	95 %	4526.5	830.8	429.0	7849.3	59.0	9270.2	5717.9	5695.2
	5 %	3297.9	0.0	0.0	0.0	21.4	161.9	0.0	0.0
J	Mean	4273.7	53.5	23.4	2586.6	48.5	2784.5	2389.7	1408.6
	95 %	4484.5	491.1	315.6	6858.1	61.1	7815.3	5090.4	5140.2
	5 %	3365.3	0.0	0.0	41.5	17.1	83.8	41.7	17.8
A	Mean	4313.2	37.7	0.0	2944.3	46.3	2979.5	2794.6	1526.4
	95 %	4512.6	466.8	0.0	7468.2	60.7	7878.2	5771.8	5080.7
	5 %	3244.5	0.0	0.0	257.5	19.4	213.3	146.4	163.2
S	Mean	3894.6	19.0	0.0	3205.2	42.6	2705.2	3343.8	1299.4
	95 %	4451.1	279.6	0.0	6819.7	57.0	6895.1	5731.7	4326.5
	5 %	3256.6	0.0	0.0	358.6	17.6	284.2	243.3	203.4
O	Mean	3659.7	19.7	12.8	2679.1	47.6	2720.9	2565.0	1286.7
	95 %	4422.7	208.7	173.0	7447.6	58.8	7984.7	5830.9	4944.4
	5 %	3267.9	0.0	0.0	559.1	26.4	321.7	339.5	337.6
N	Mean	3681.7	2.8	0.0	2697.0	45.7	2890.7	2872.1	991.5
	95 %	4461.4	37.7	0.0	5788.2	56.9	6456.3	5532.7	2939.9
	5 %	3267.8	0.0	0.0	433.7	22.1	562.9	260.8	301.4
D	Mean	3785.0	30.8	49.3	3046.0	45.5	3174.0	3137.2	1254.1
	95 %	4435.5	395.4	531.8	6774.4	58.8	7417.7	6016.2	3941.2



<u>Figure 15:</u> Simulation Results — Experiment I

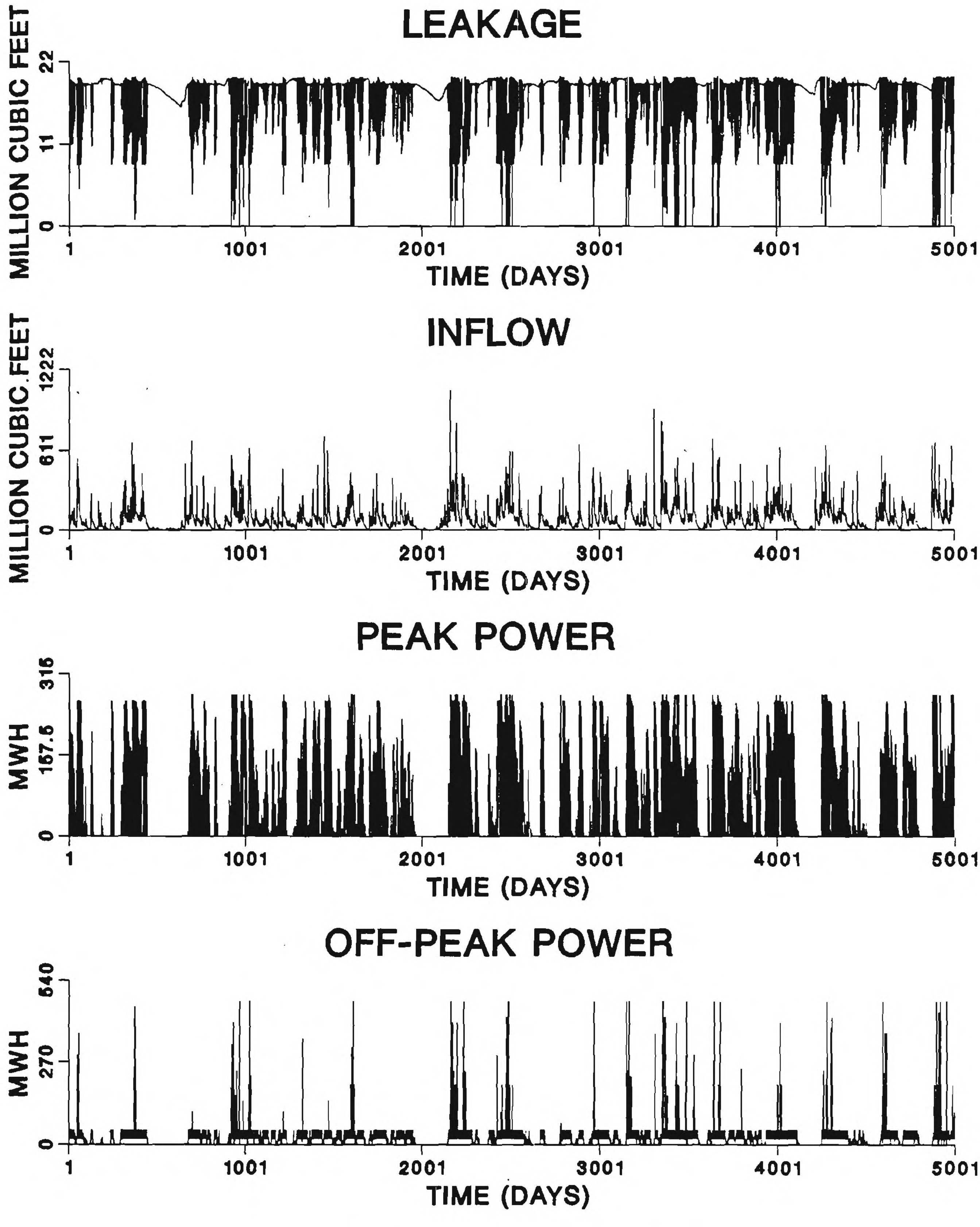


Figure 16: Simulation Results — Experiment I

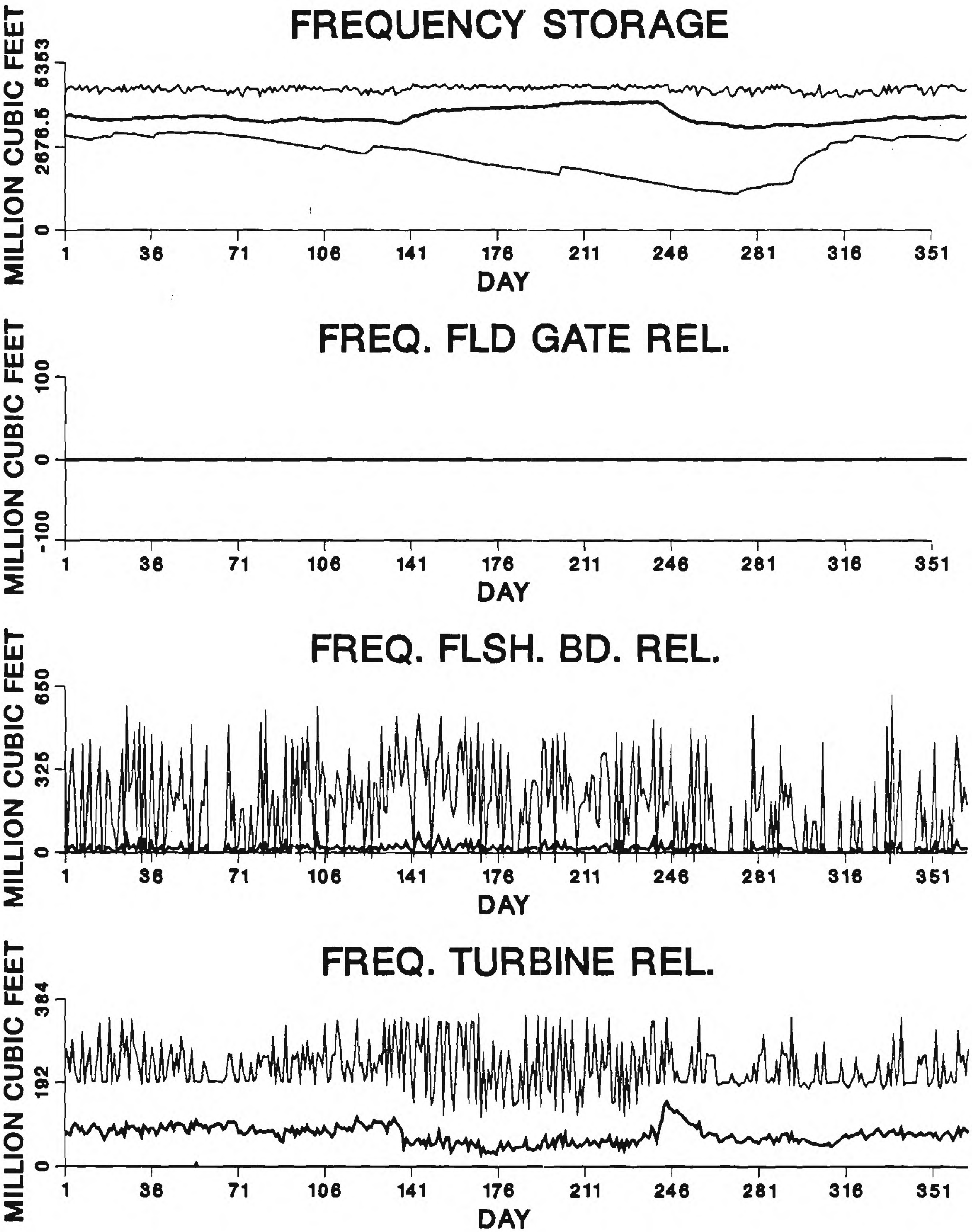


Figure 17: Daily Frequencies — Experiment I

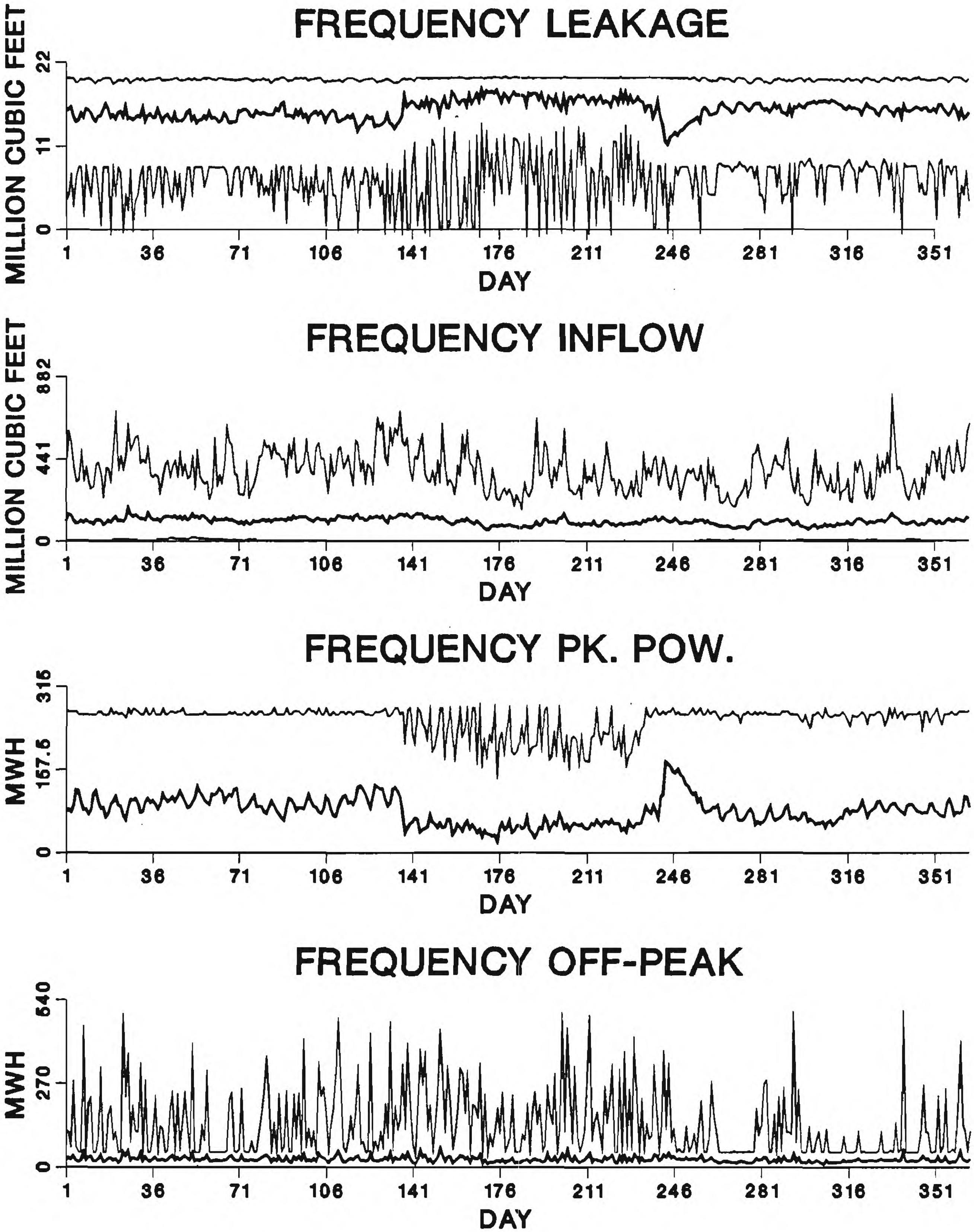
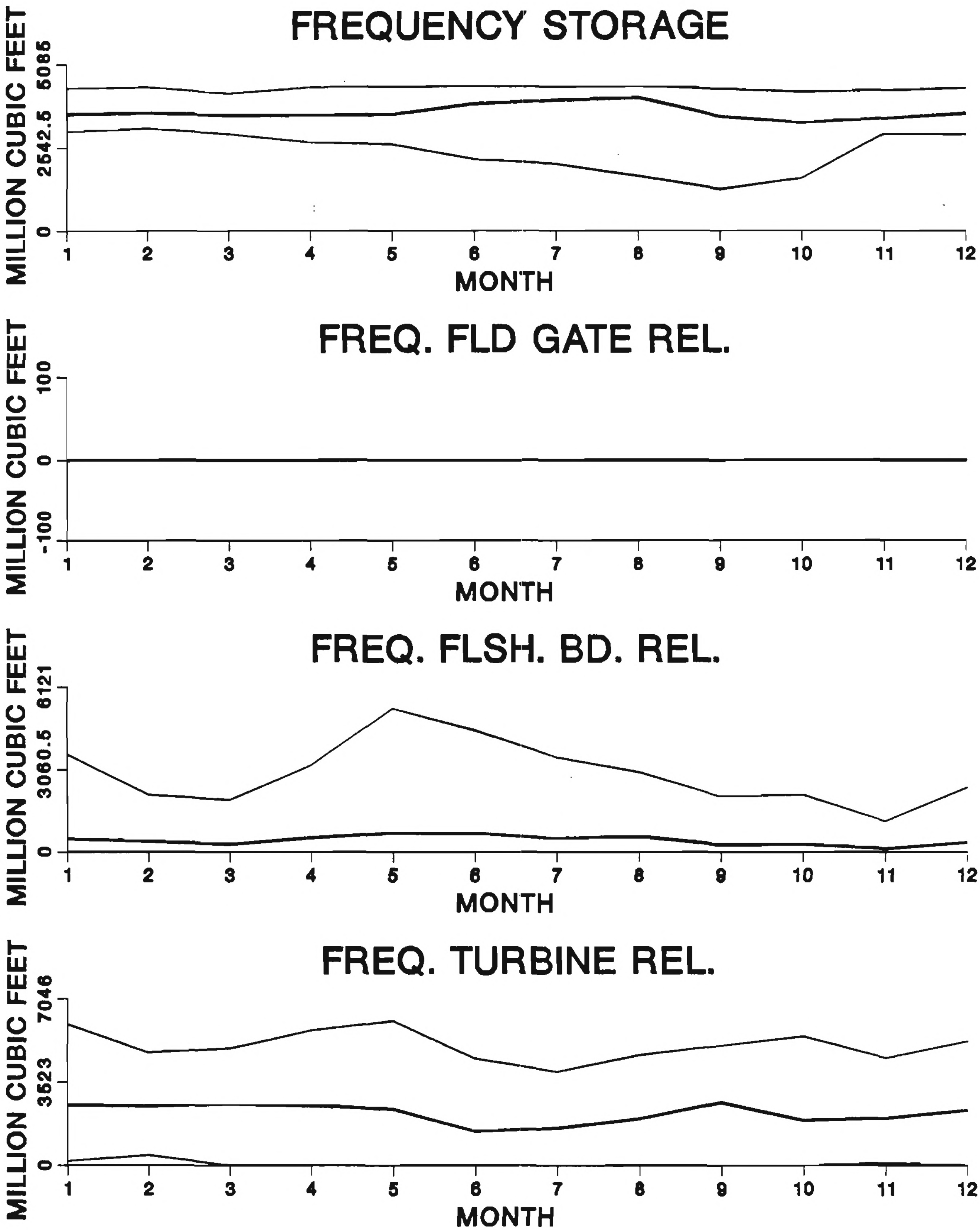


__Figure 18:__ Daily Frequencies — Experiment I



<u>Figure 19:</u> Monthly Frequencies — Experiment I

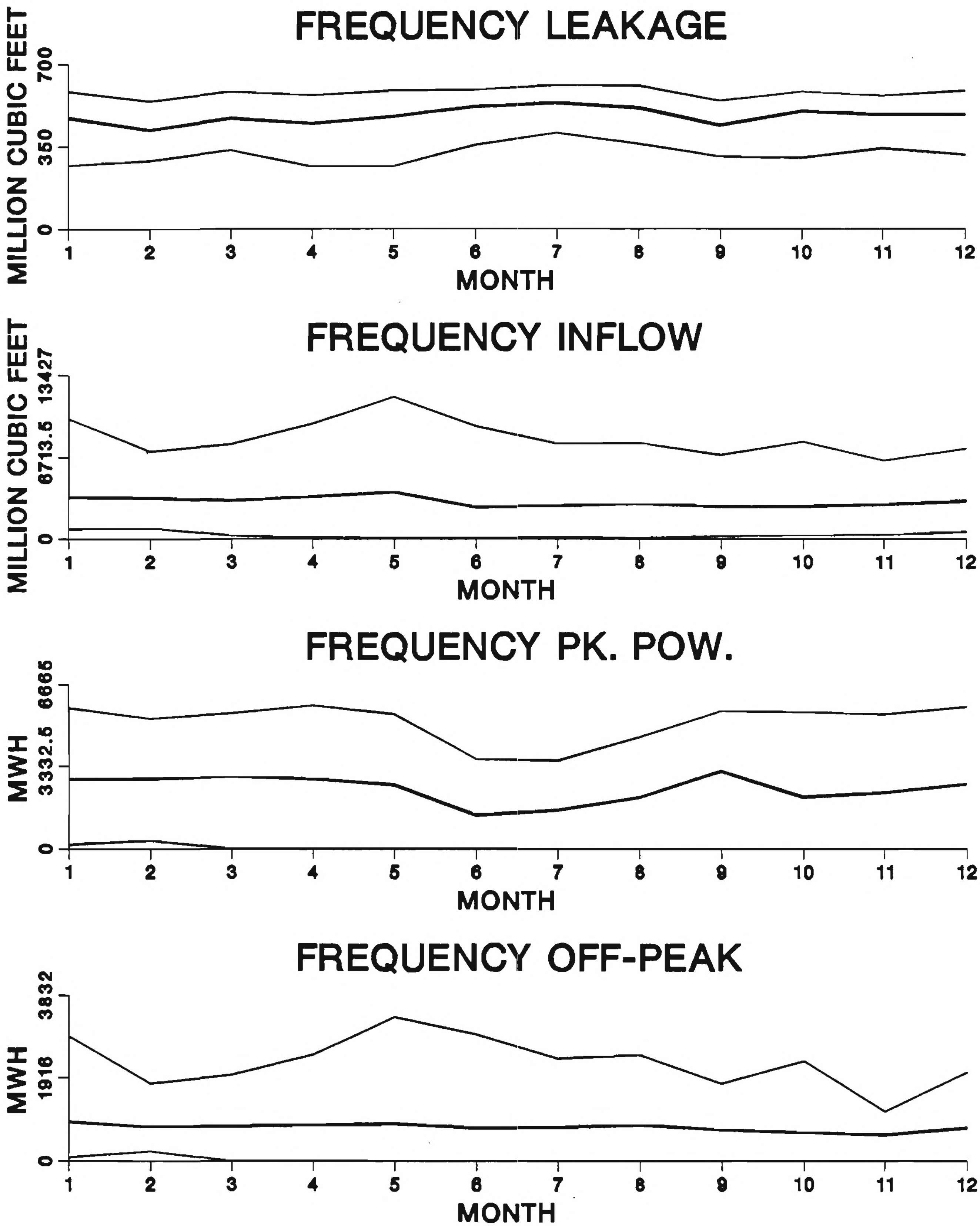


Figure 20: Monthly Frequencies — Experiment I

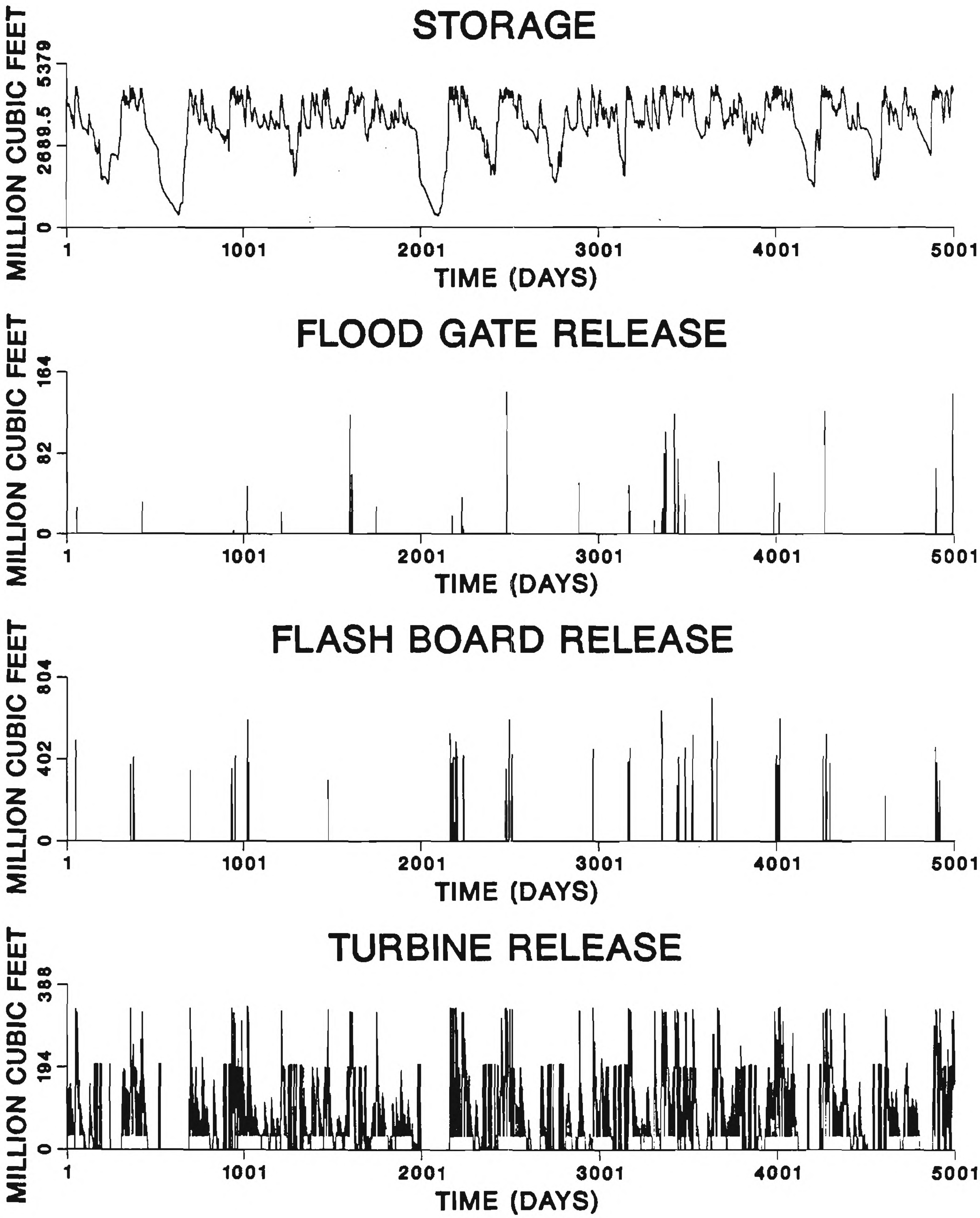


Figure 21: Simulation Results — Experiment II

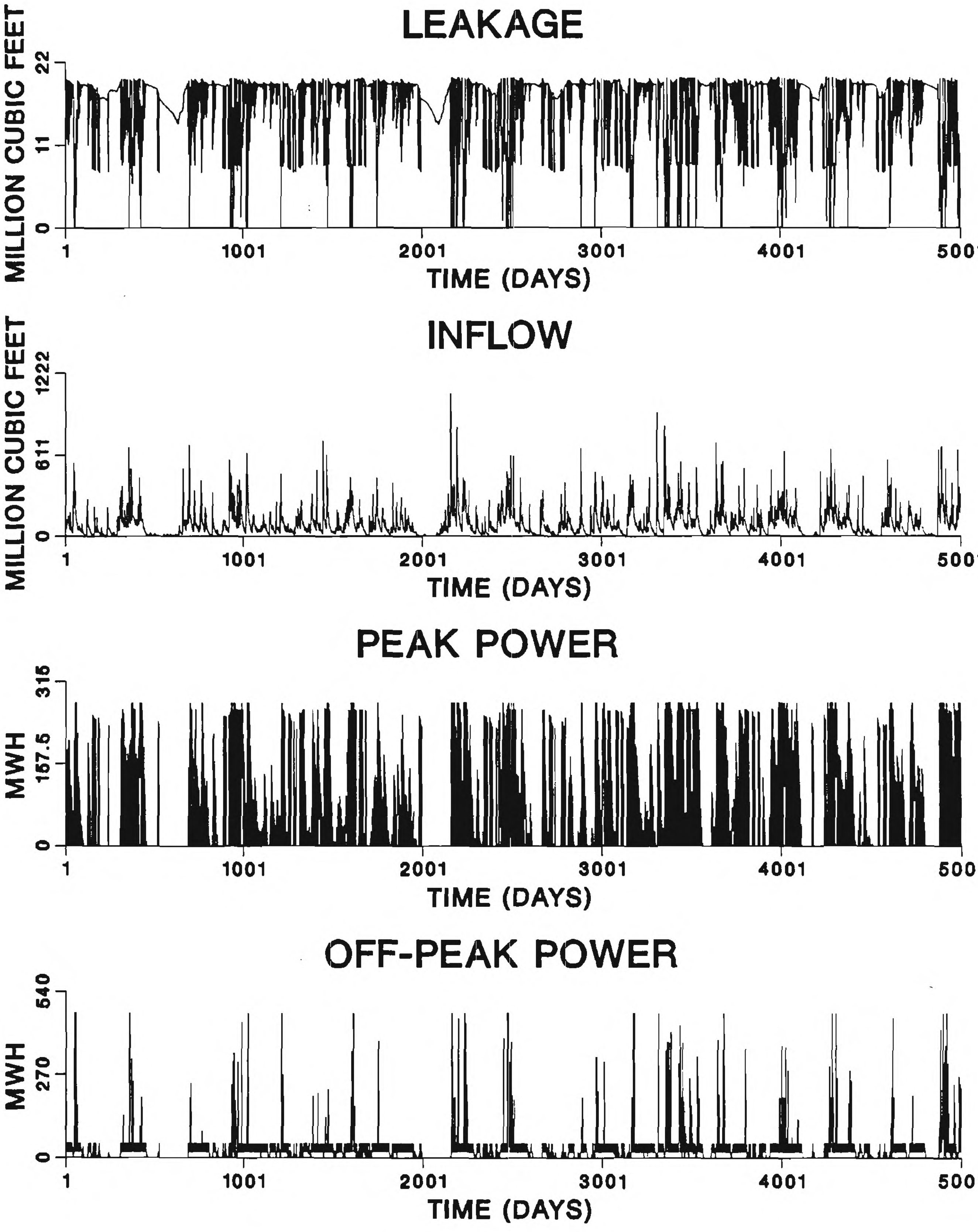


Figure 22: Simulation Results — Experiment II

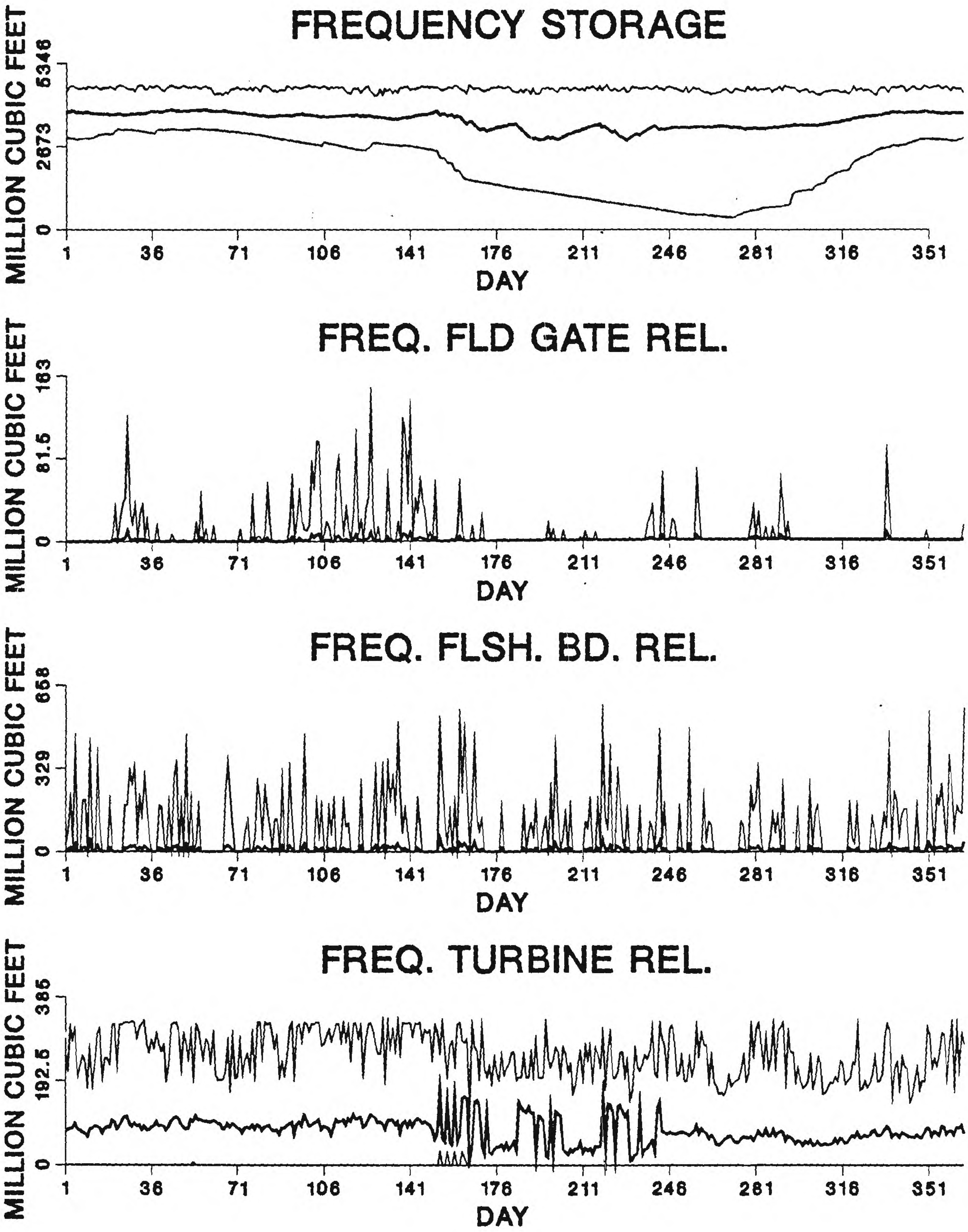
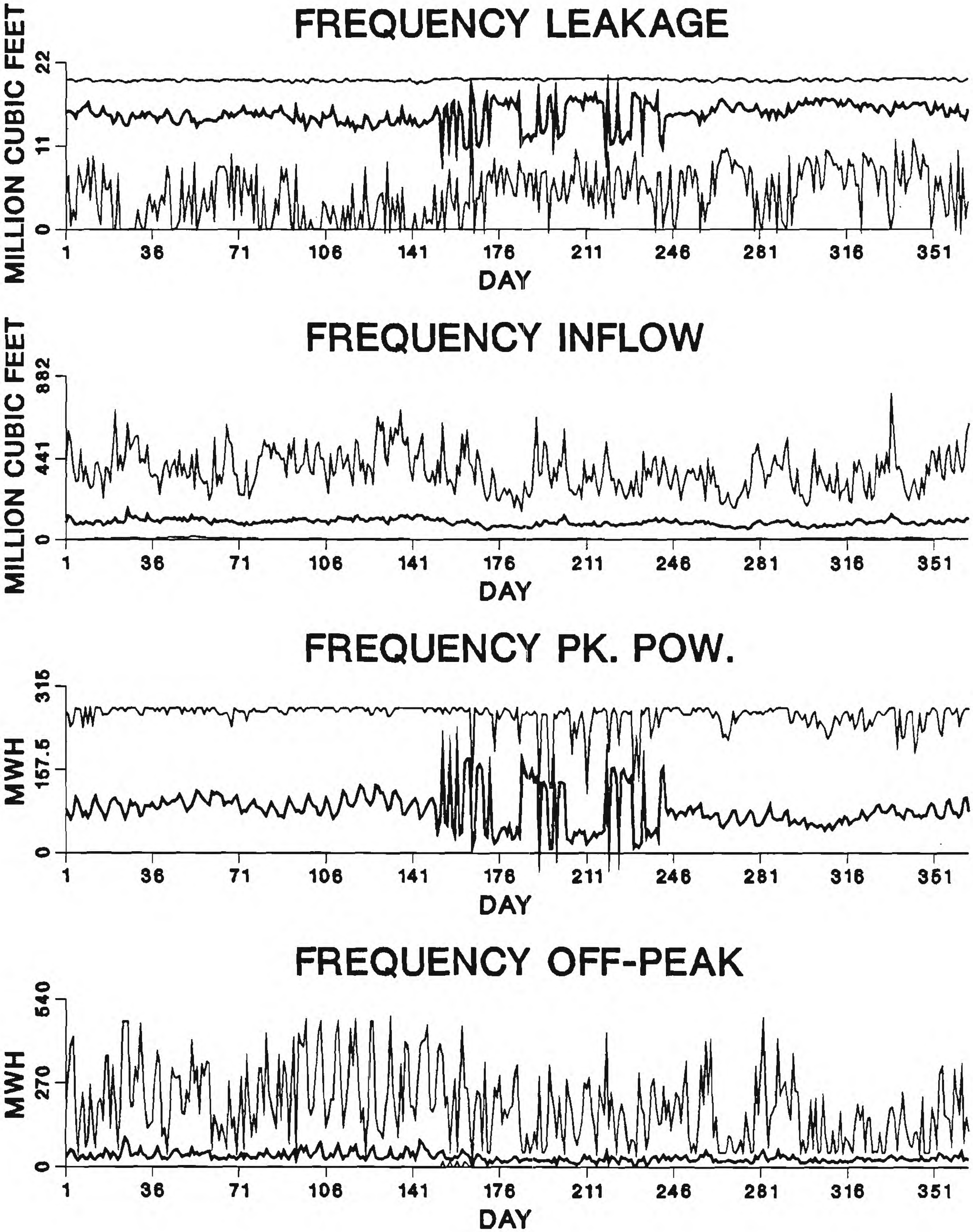


Figure 23: Daily Frequencies — Experiment II



<u>Figure 24:</u> Daily Frequencies — Experiment II

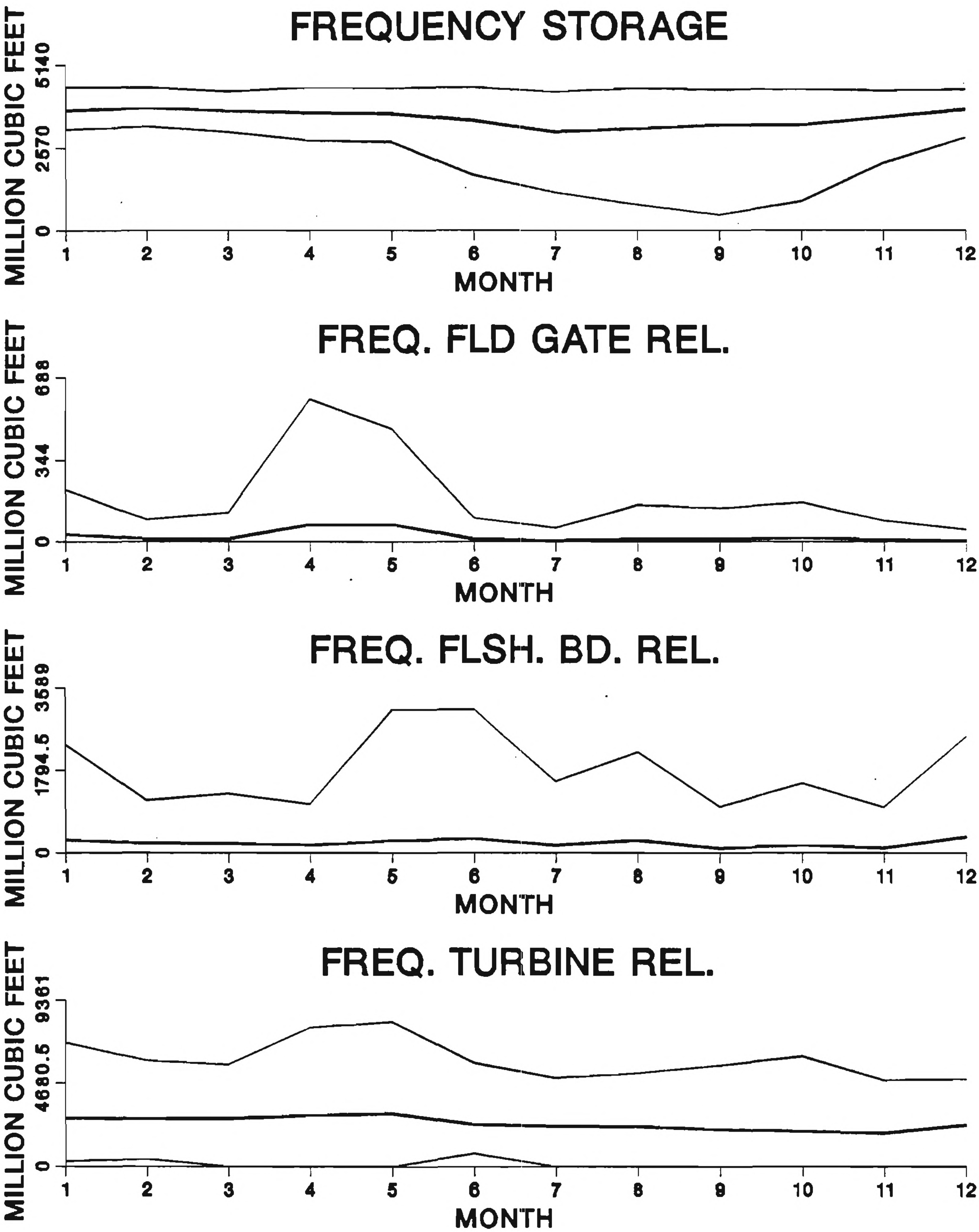


Figure 25: Monthly Frequencies — Experiment II

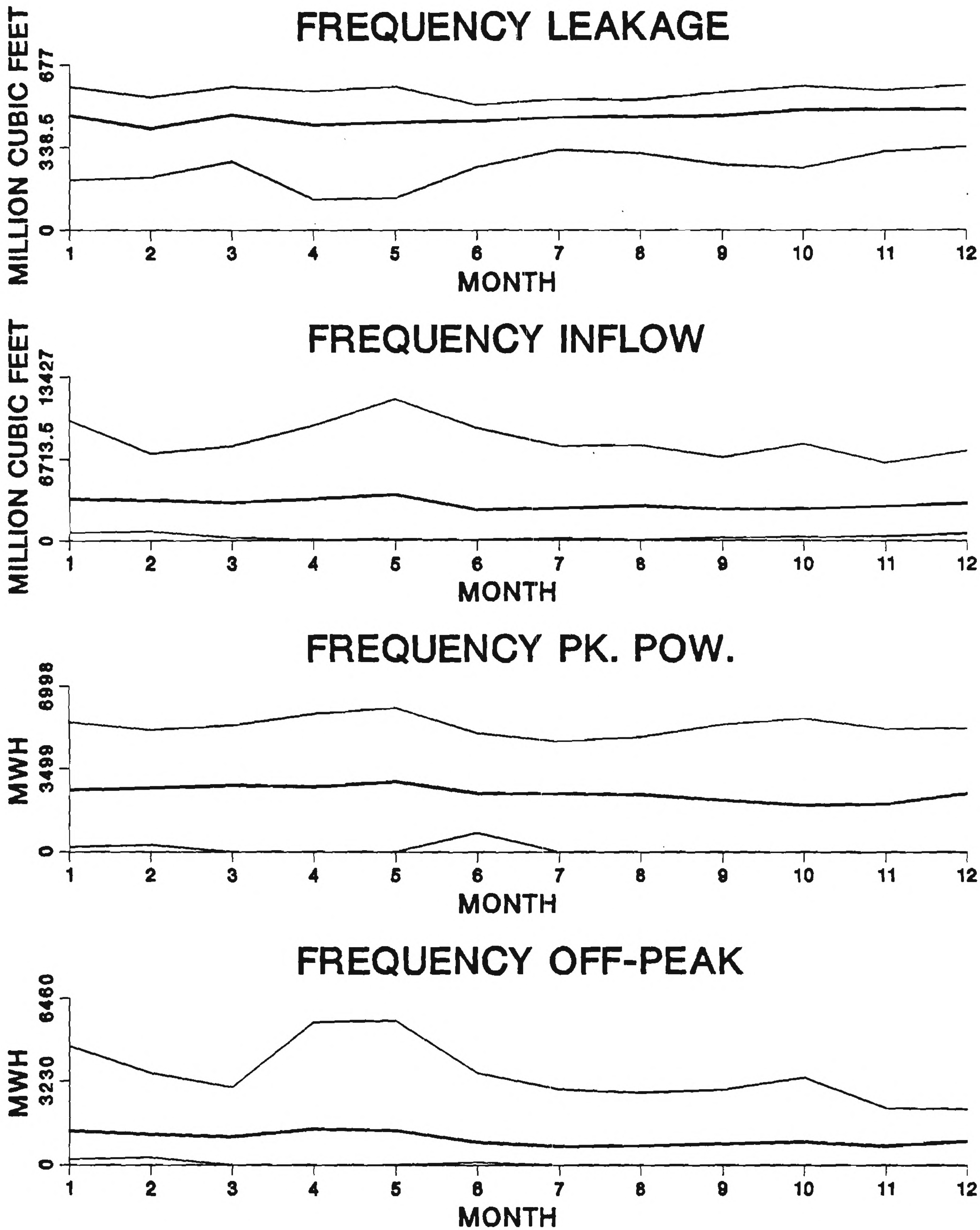
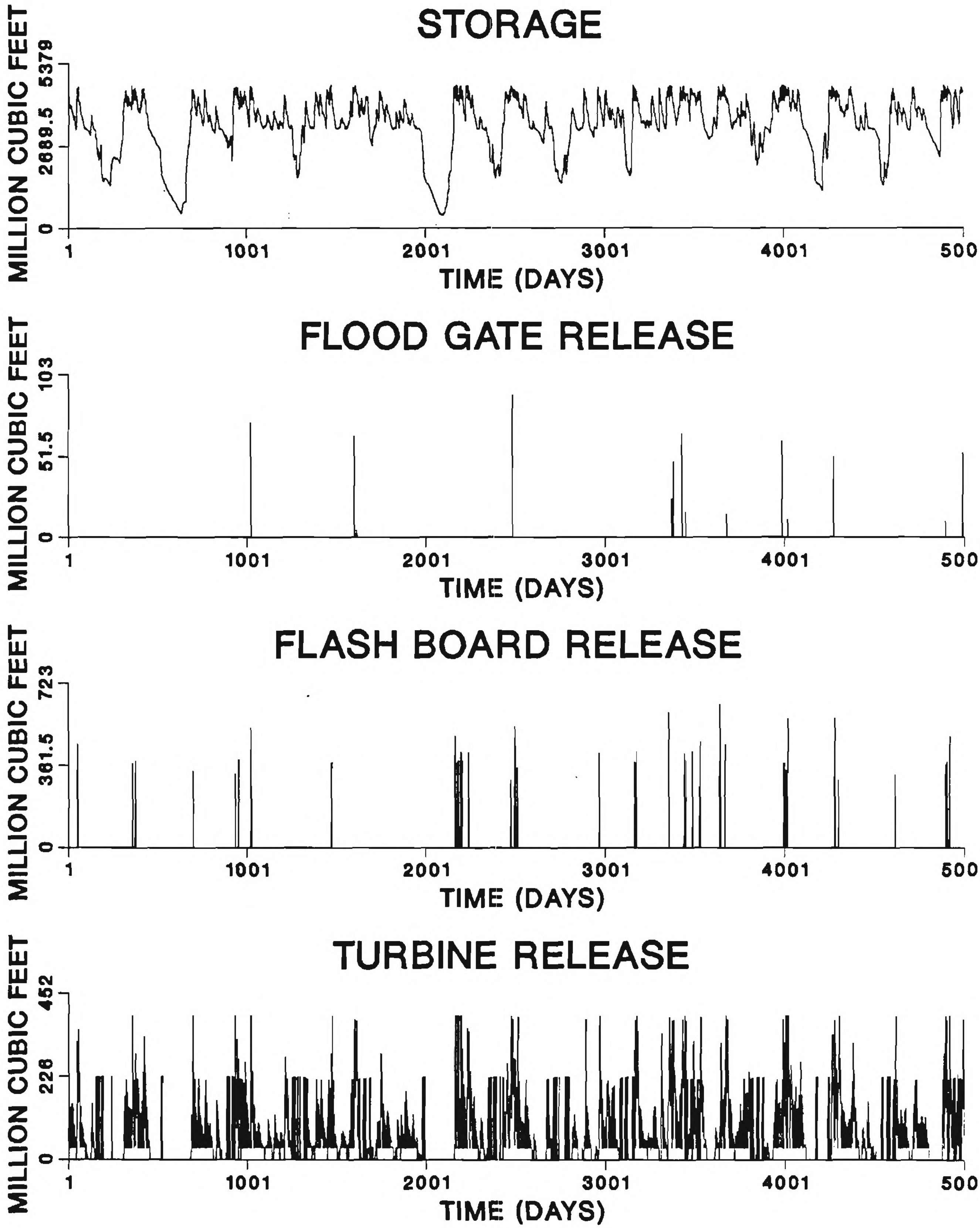


Figure 26: Monthly Frequencies — Experiment II



<u>Figure 27:</u> Simulation Results — Experiment III

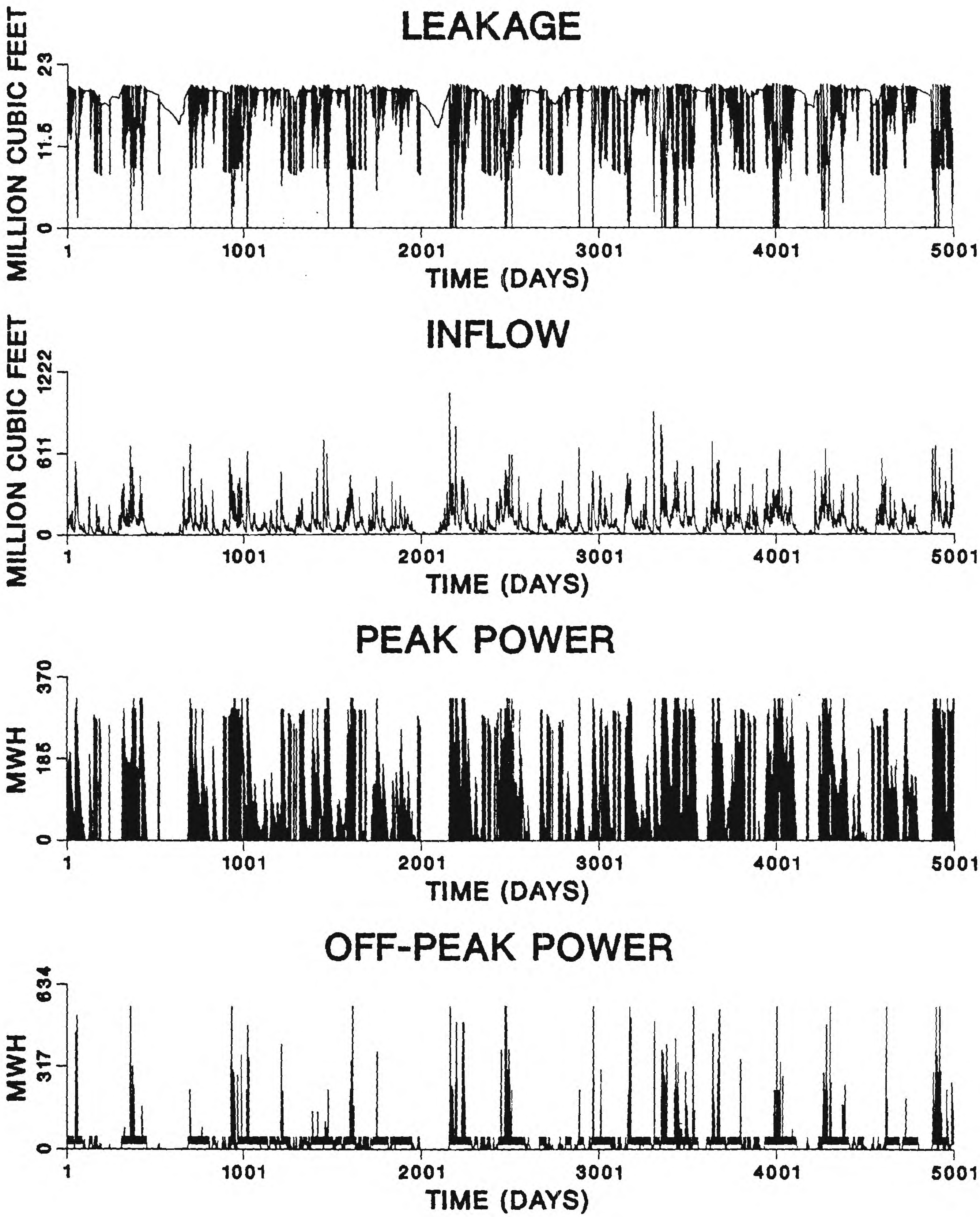


Figure 28: Simulation Results — Experiment III

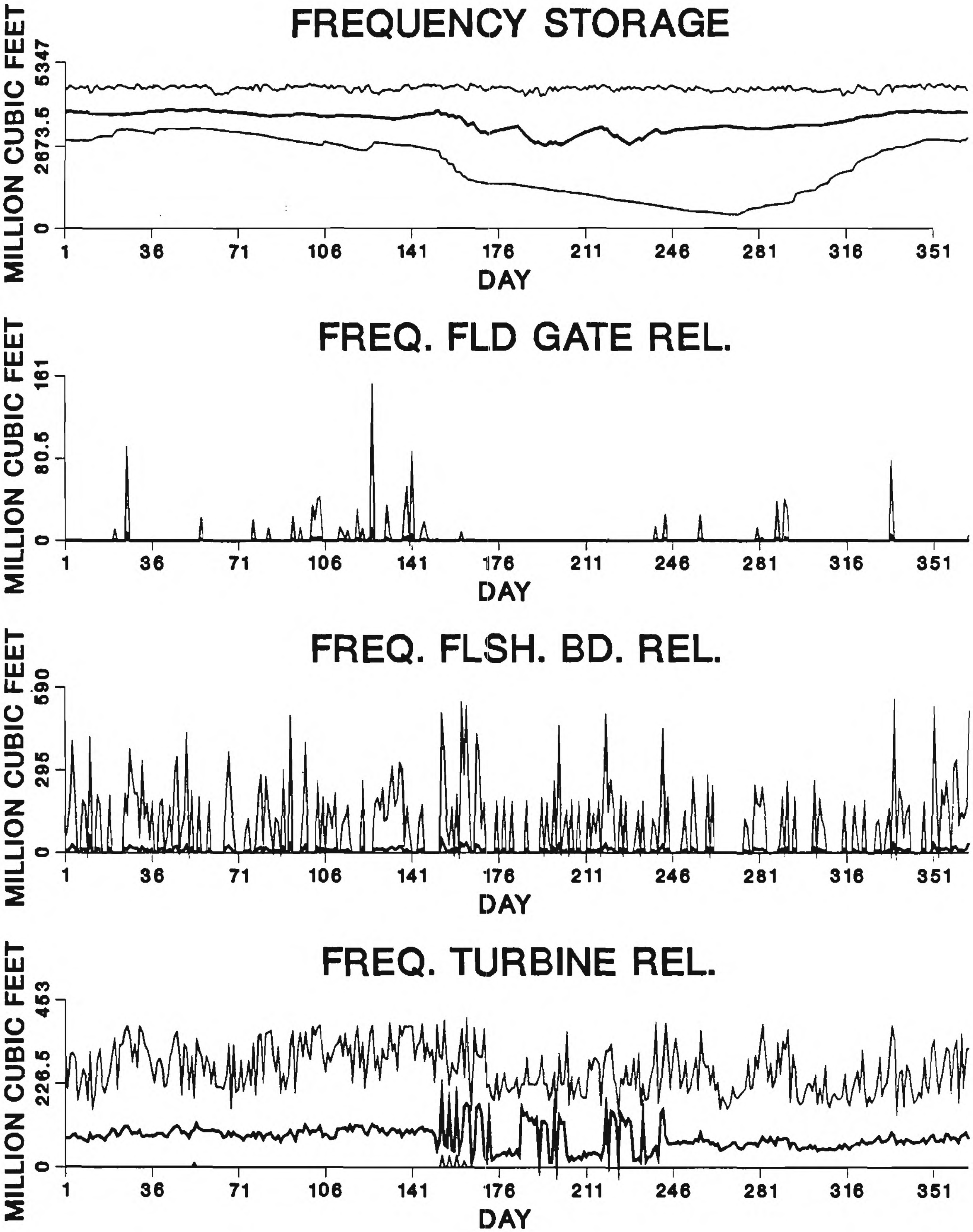


Figure 29: Daily Frequencies — Experiment III

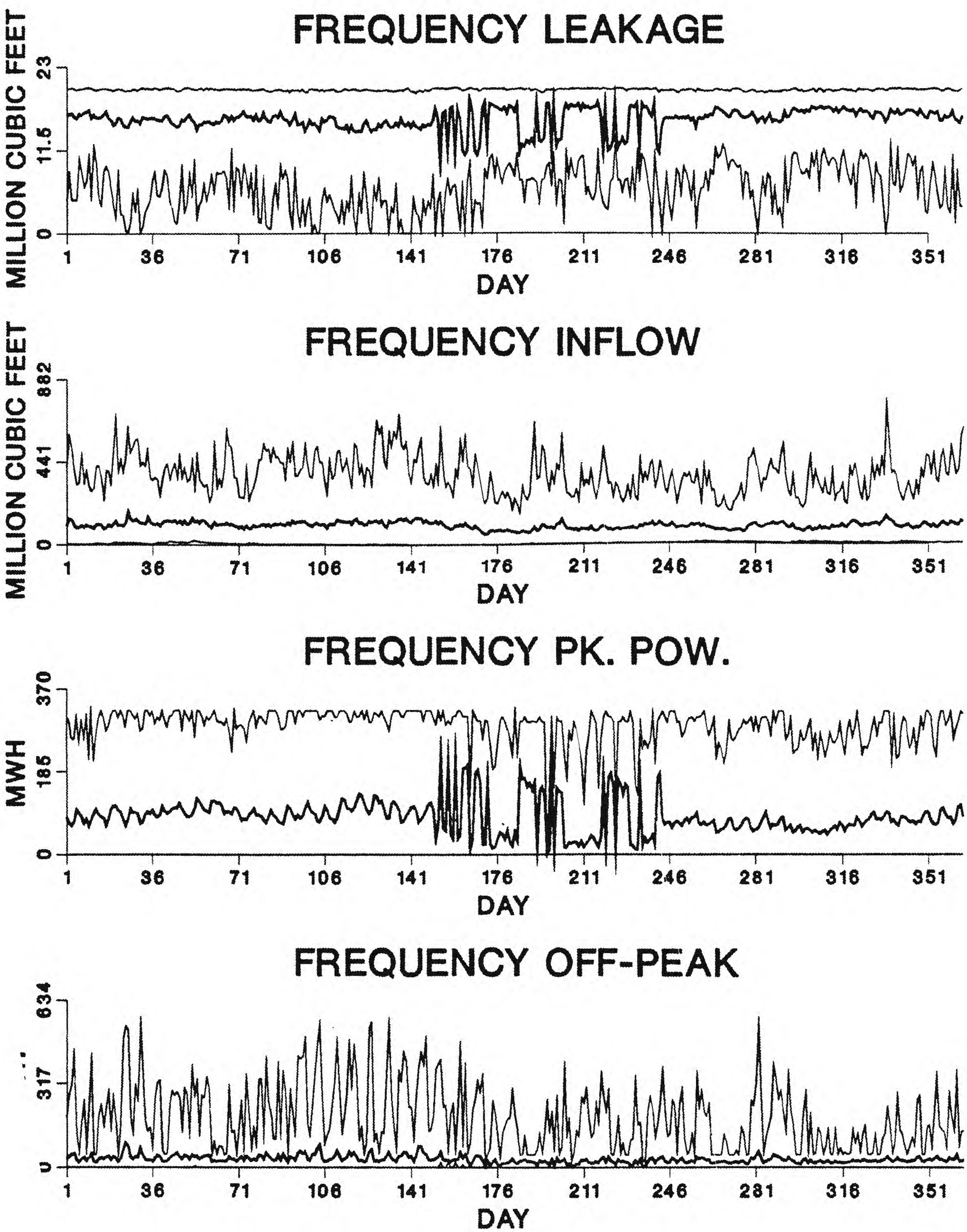


Figure 30: Daily Frequencies — Experiment III

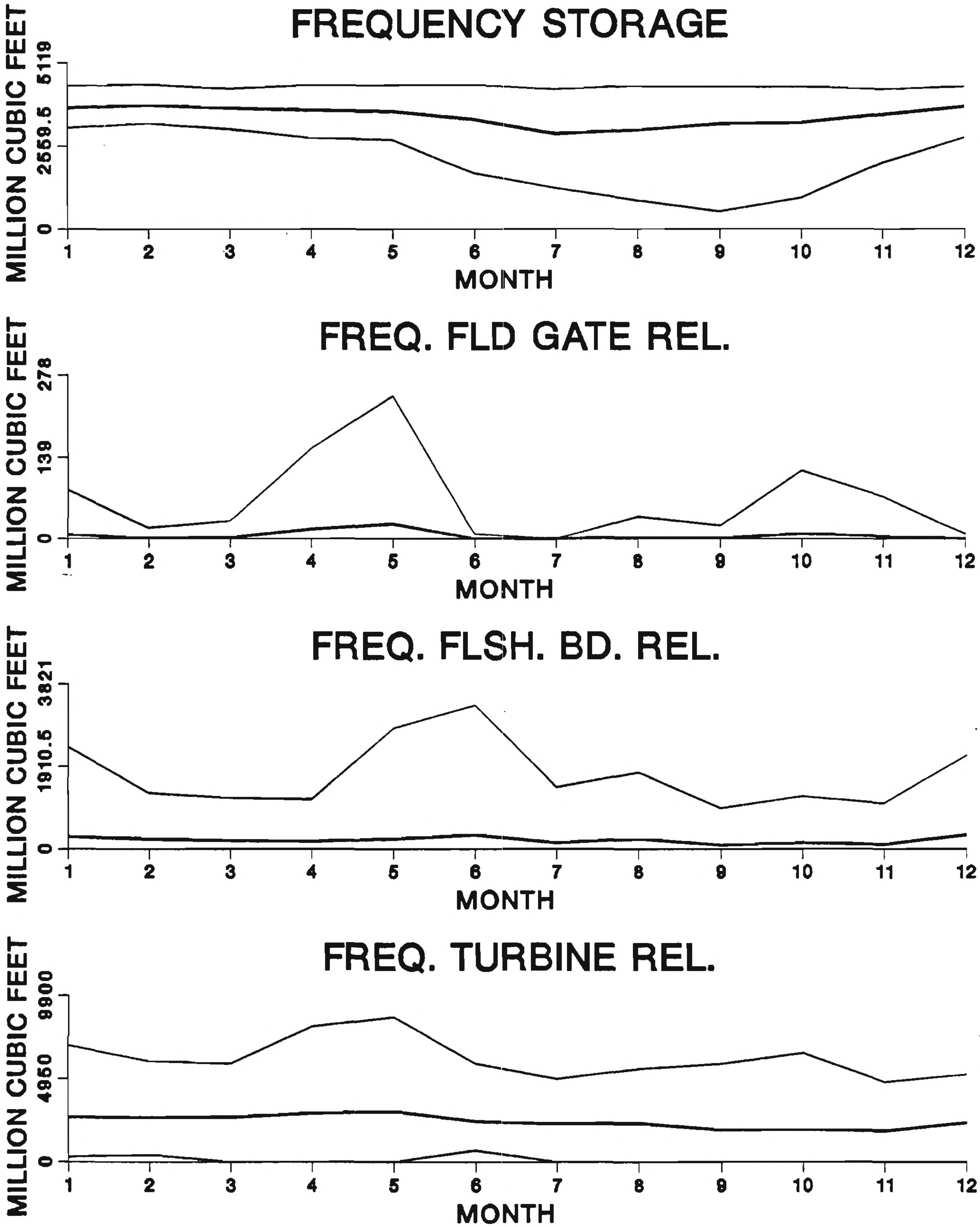
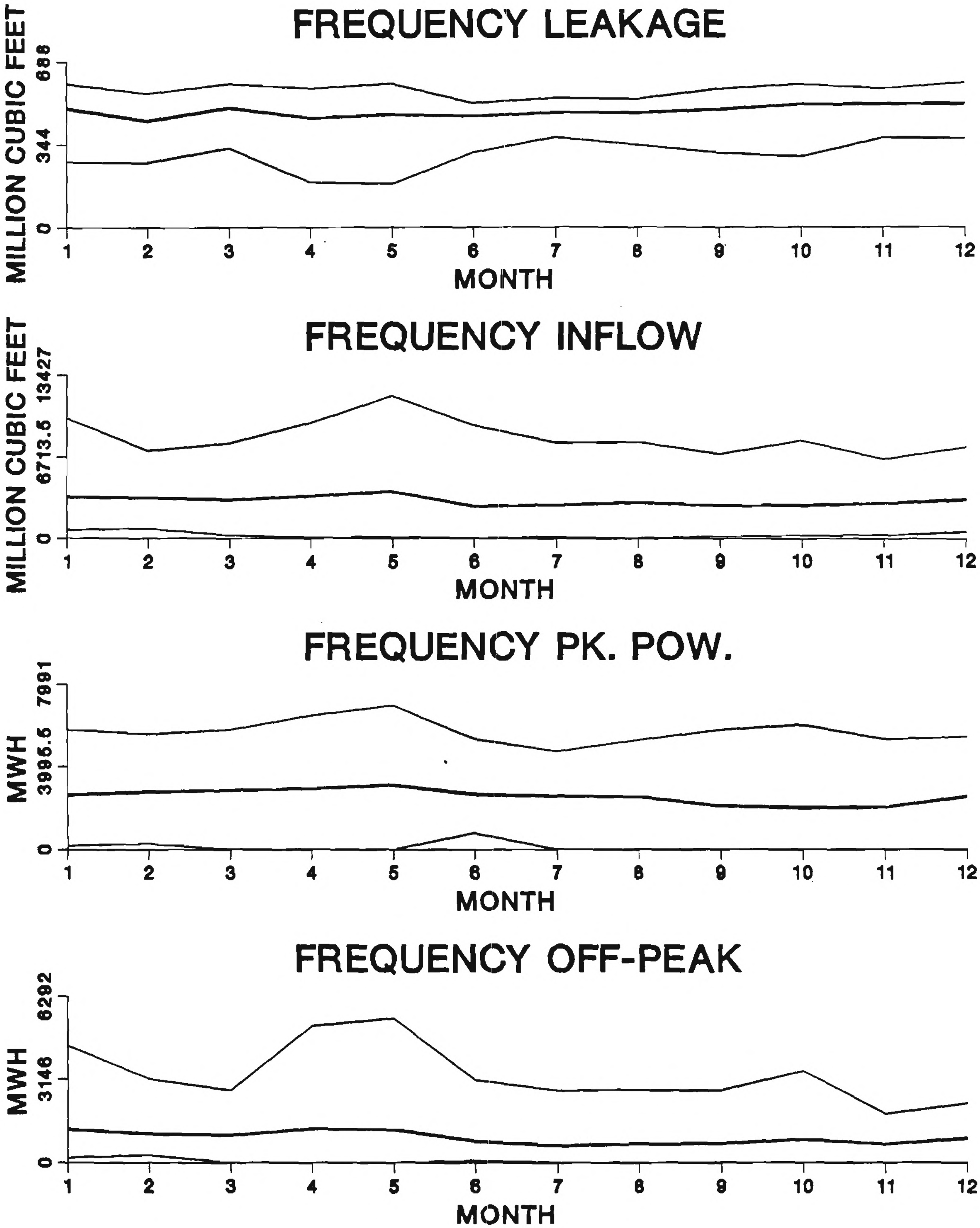


Figure 31: Monthly Frequencies — Experiment III



<u>Figure 32:</u> Monthly Frequencies — Experiment III

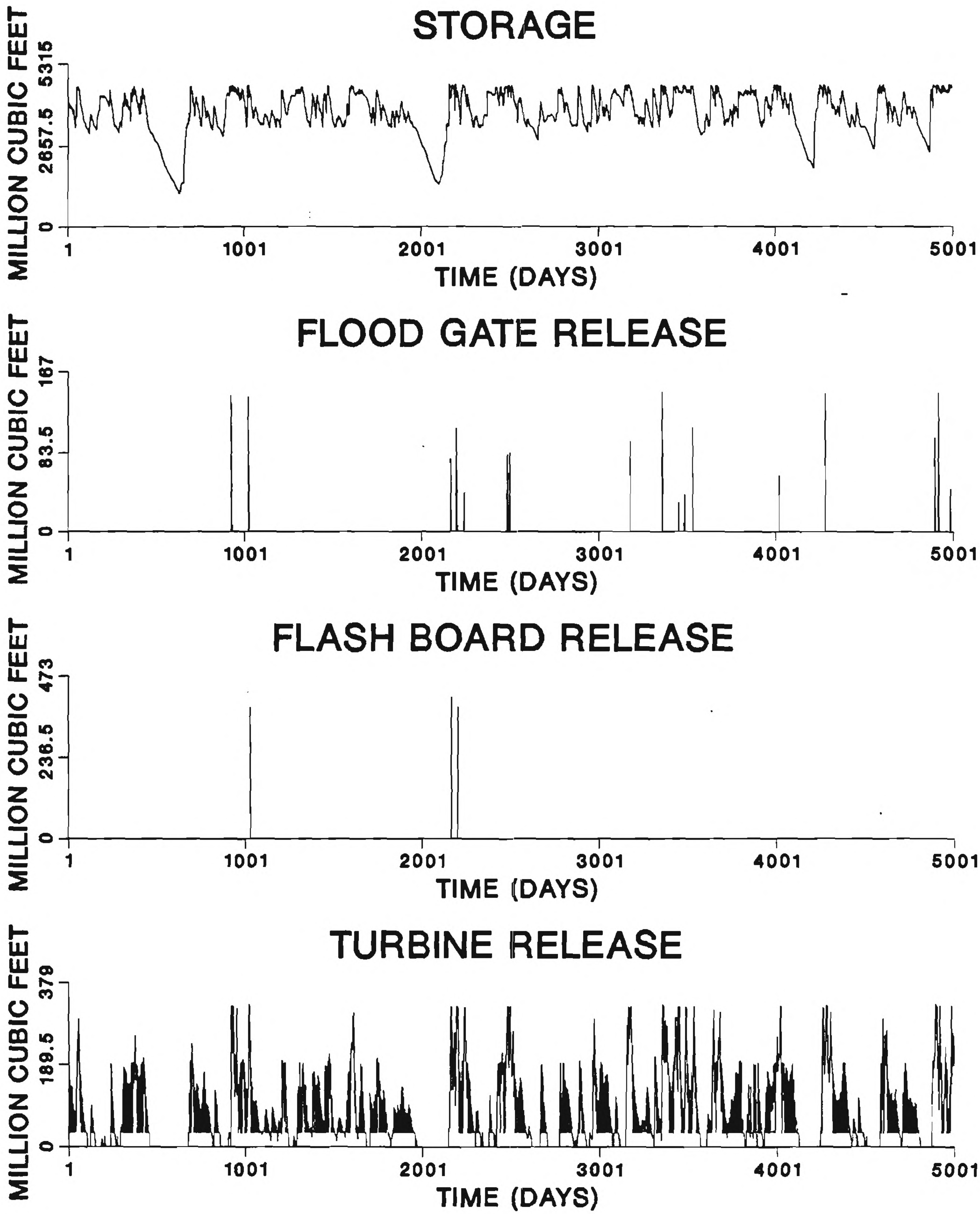


Figure 33: Simulation Results — Experiment IV

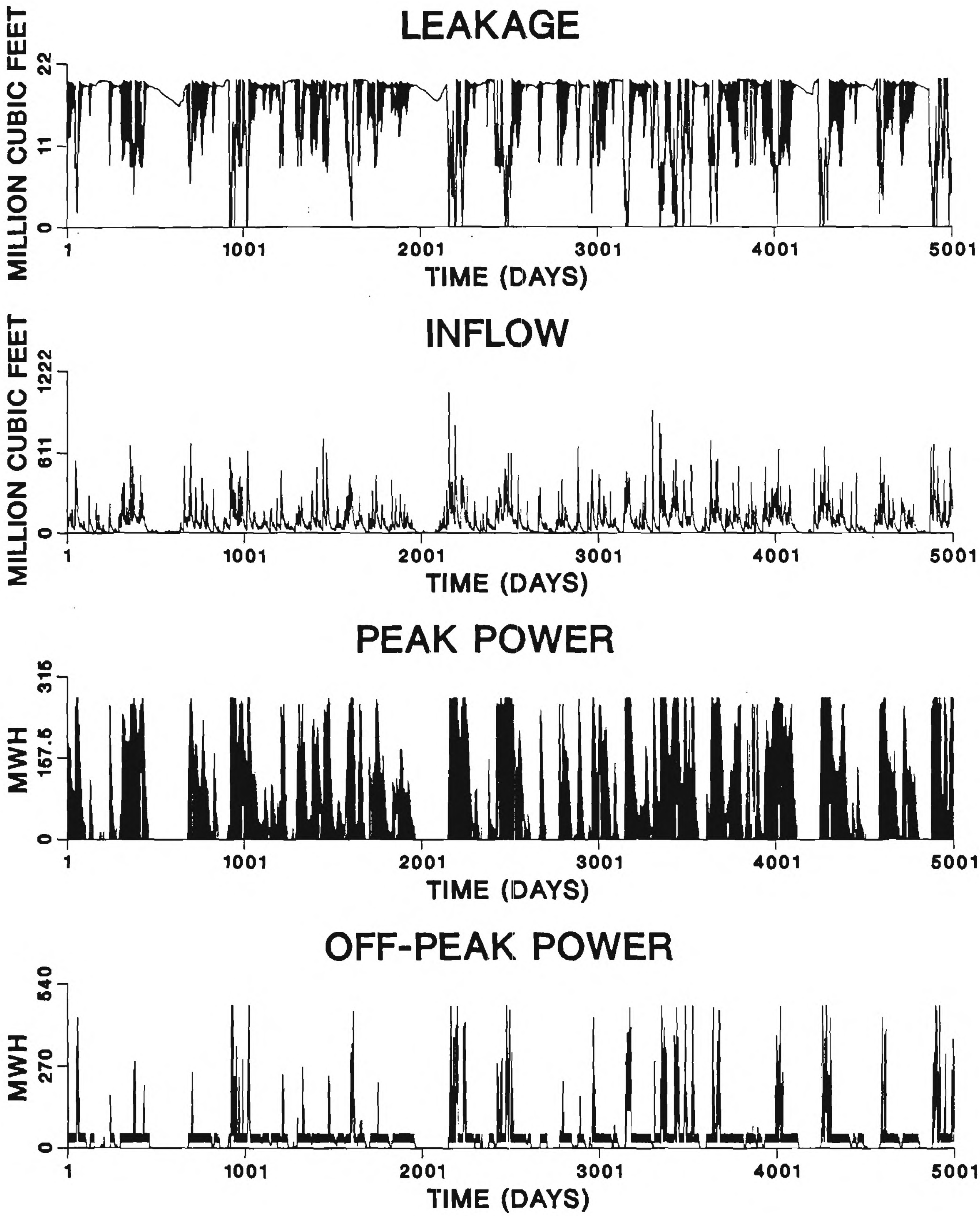


Figure 34: Simulation Results — Experiment IV

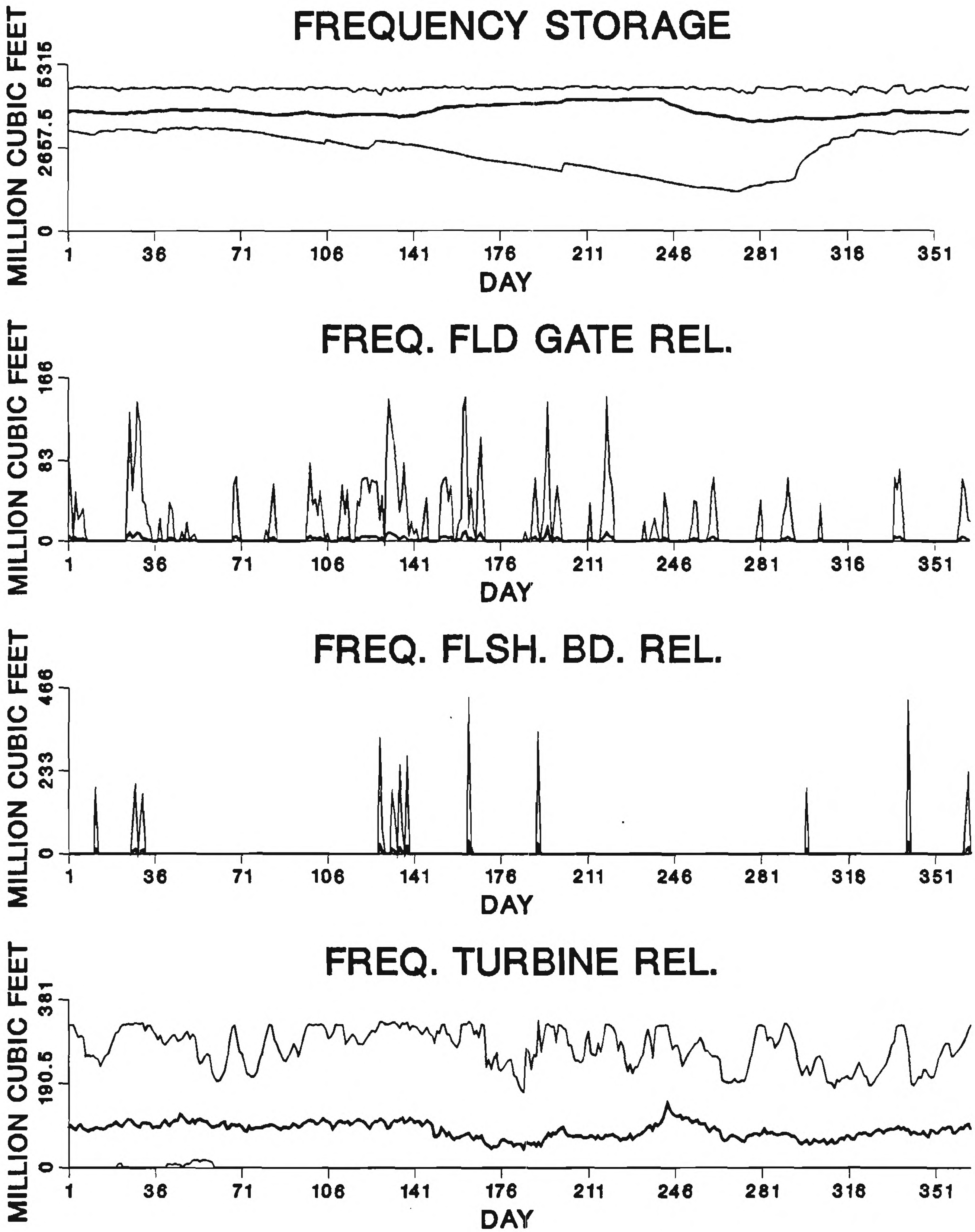


Figure 35: Daily Frequencies — Experiment IV

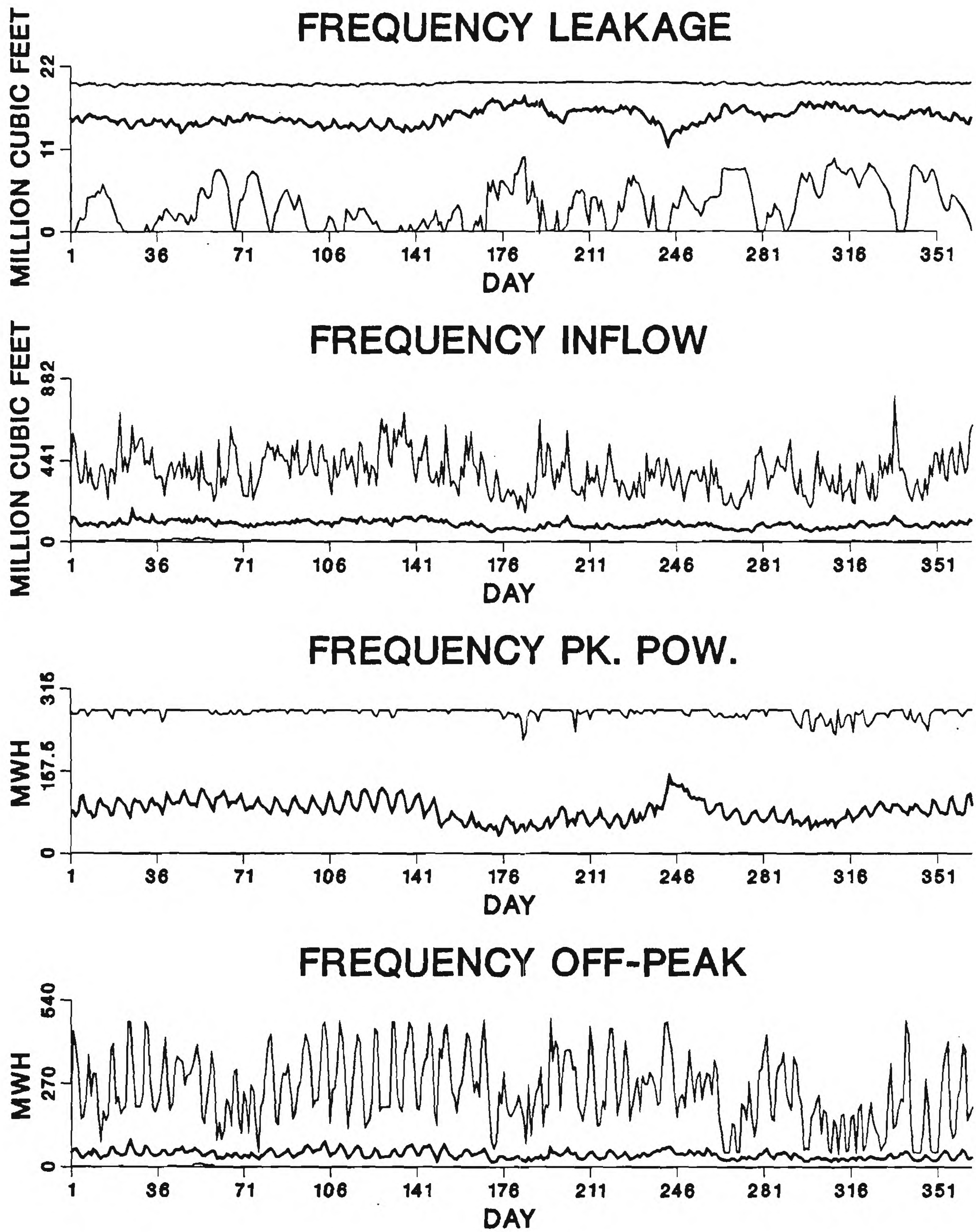


Figure 36: Daily Frequencies — Experiment IV

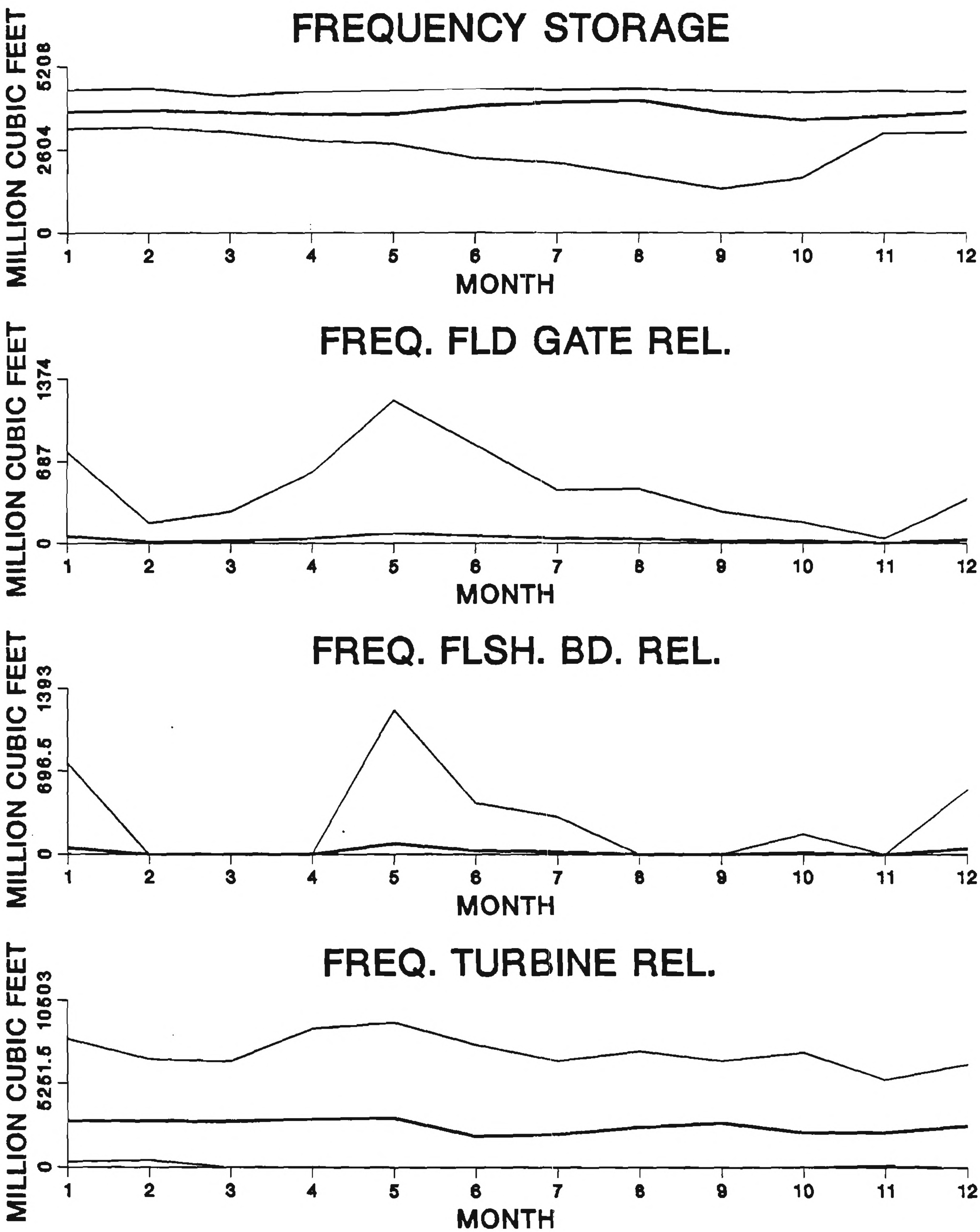
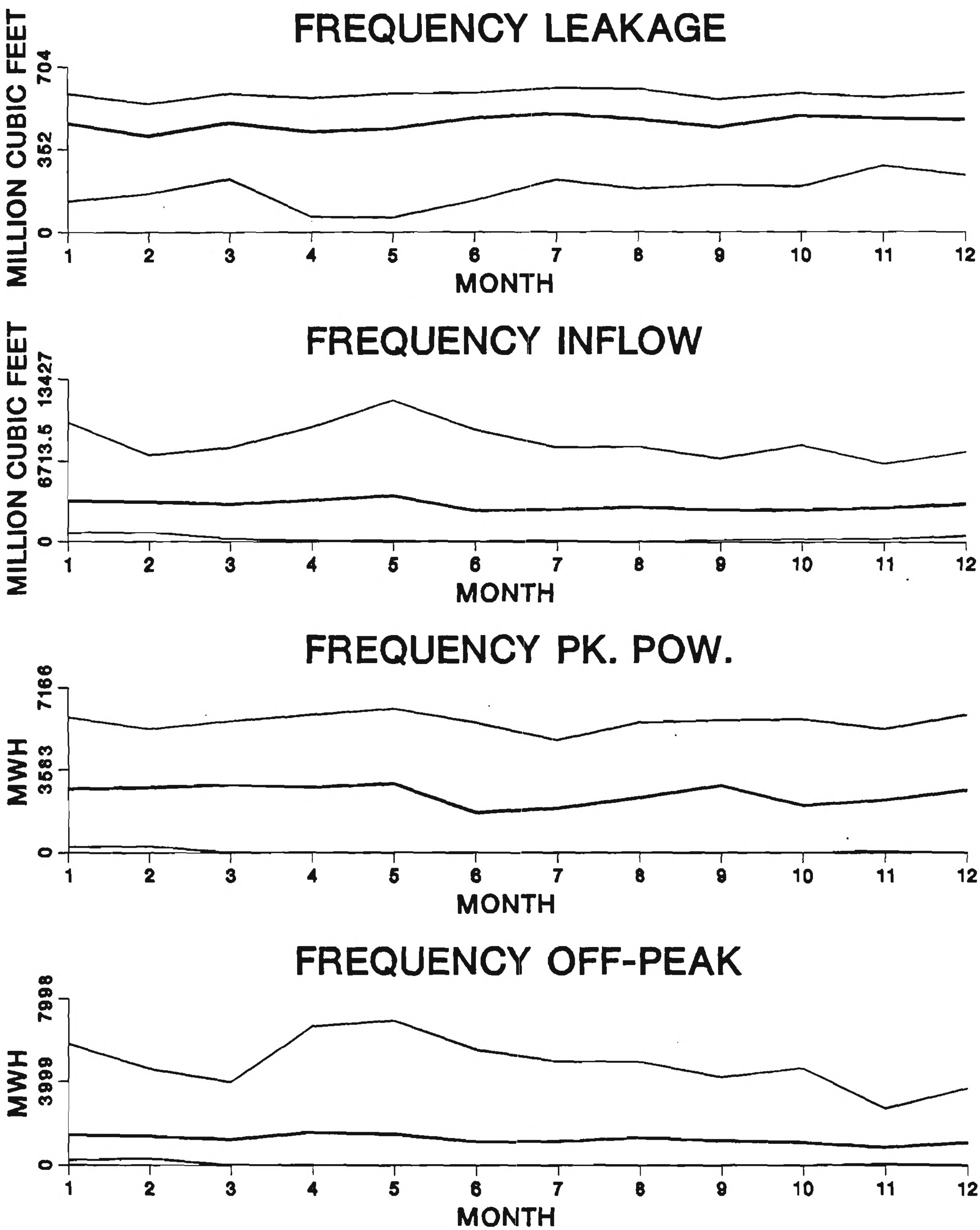


Figure 37: Monthly Frequencies — Experiment IV



<u>Figure 38:</u> Monthly Frequencies — Experiment IV

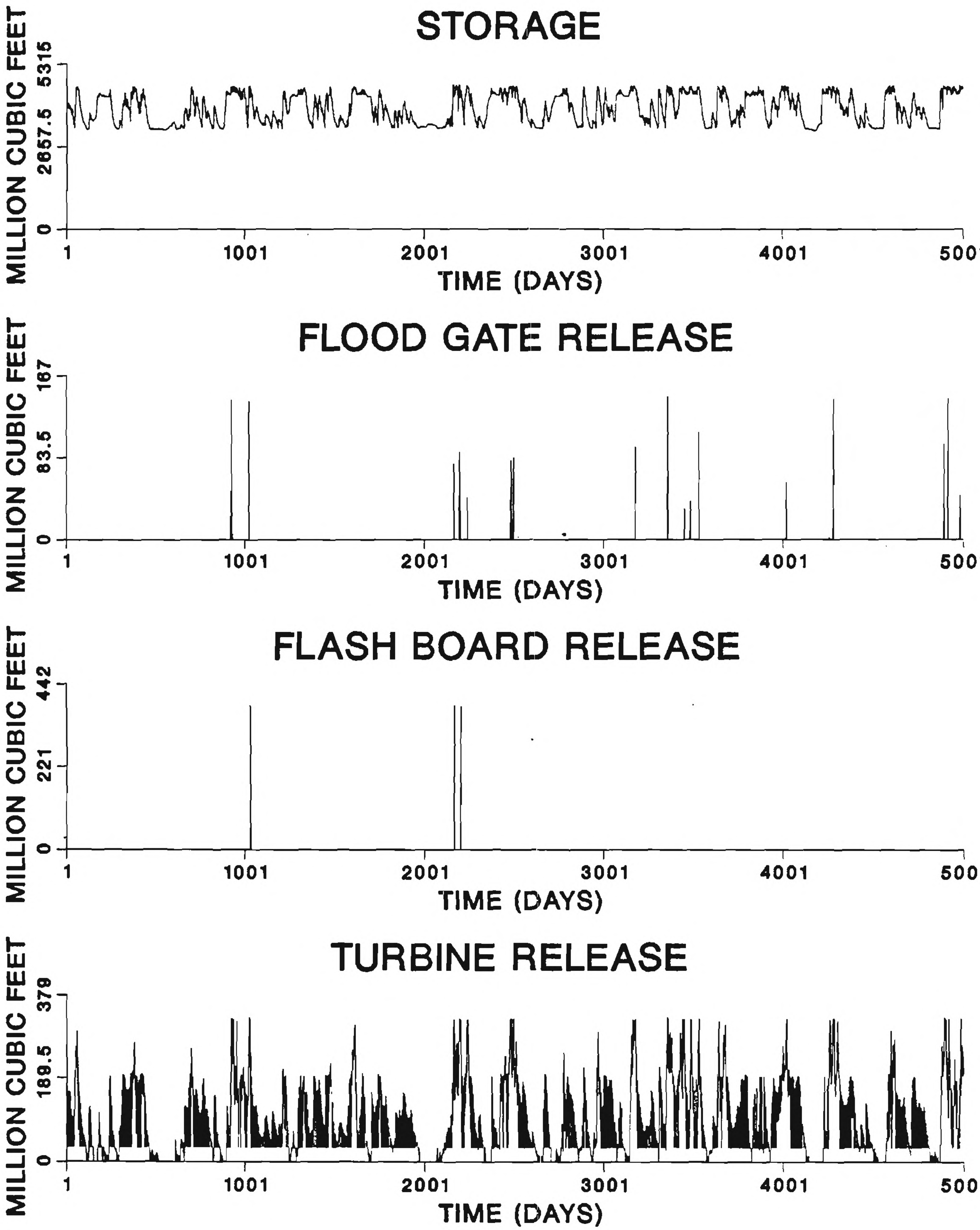


Figure 39: Simulation Results — Experiment V

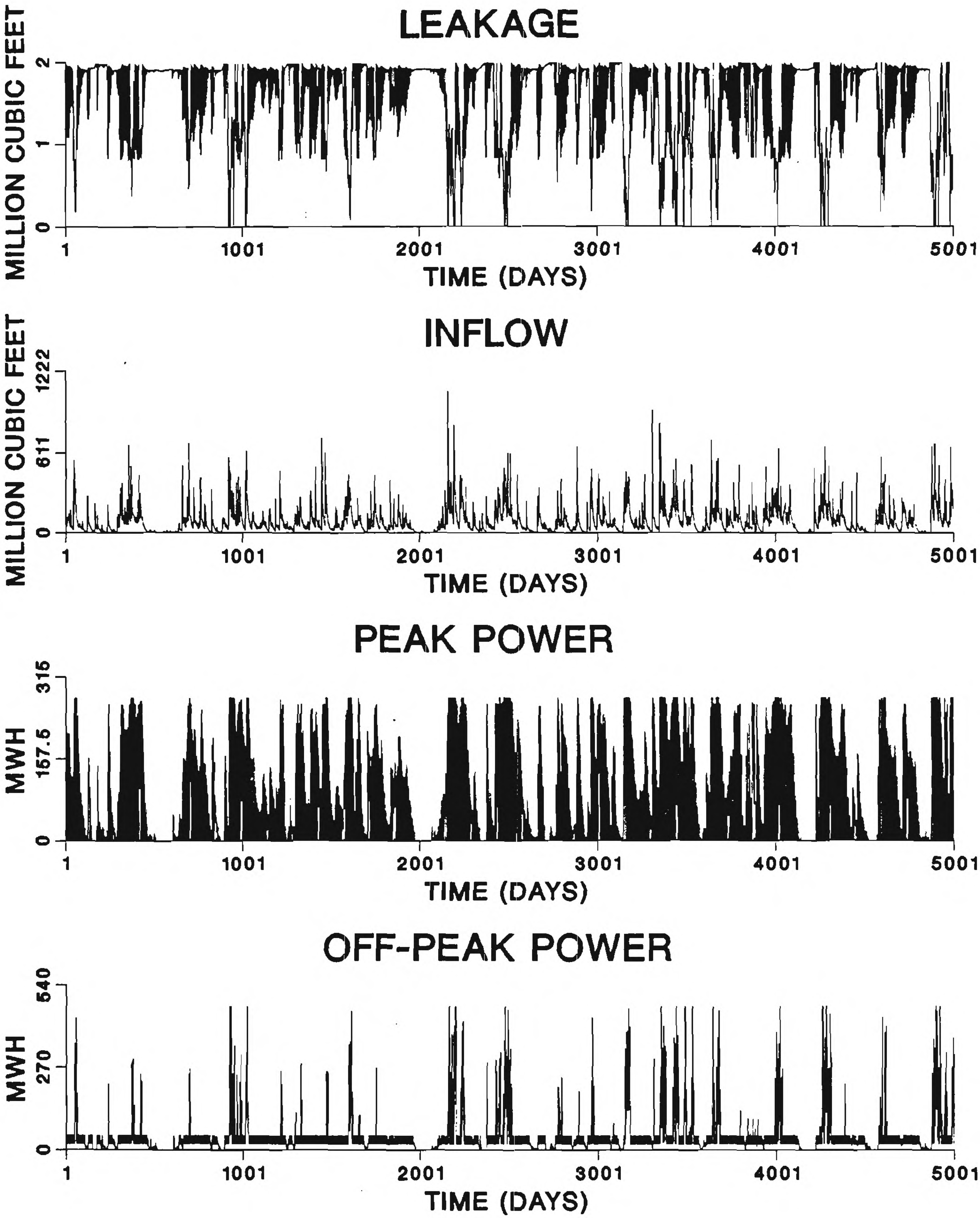


Figure 40: Simulation Results — Experiment V

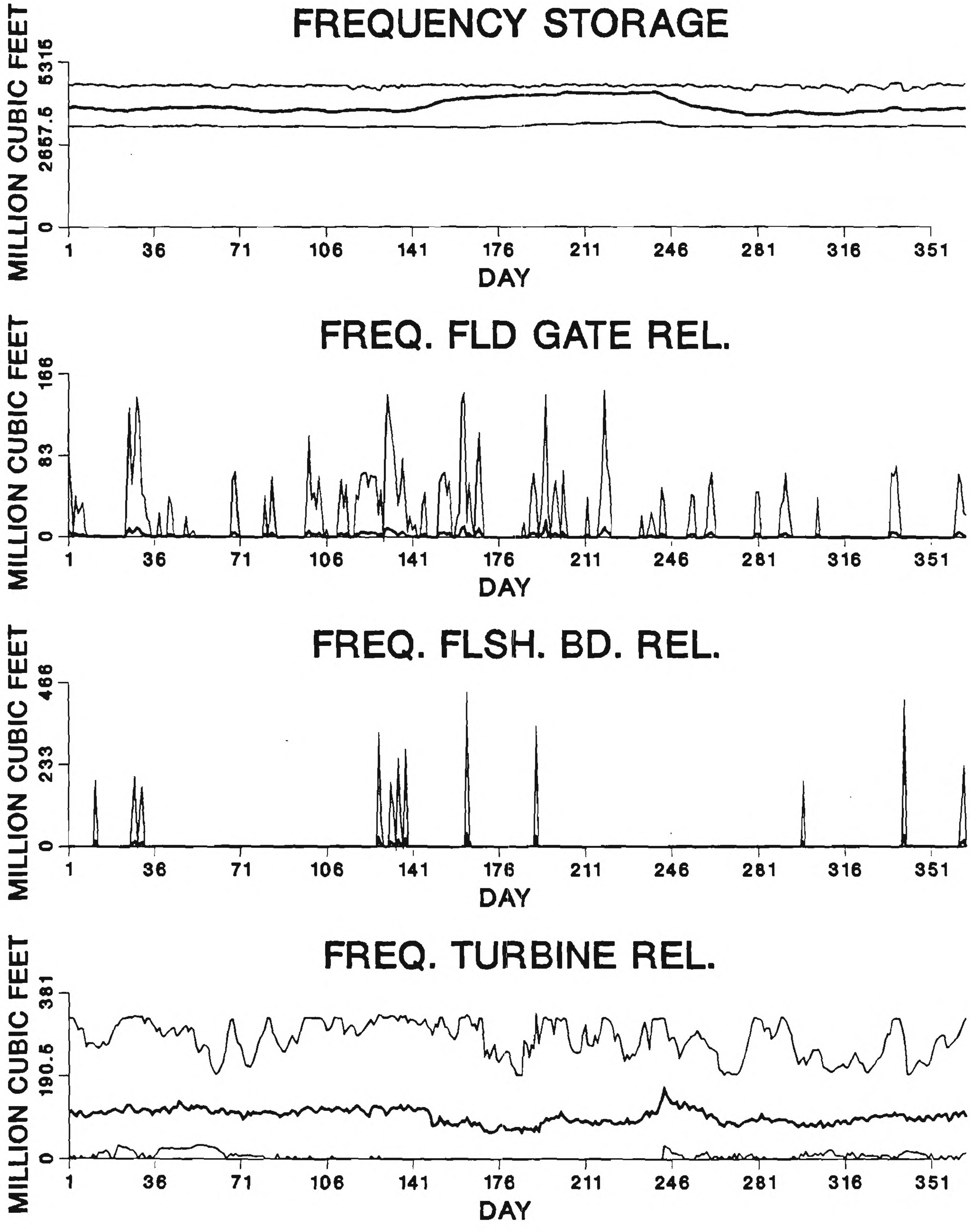


Figure 41: Daily Frequencies — Experiment V

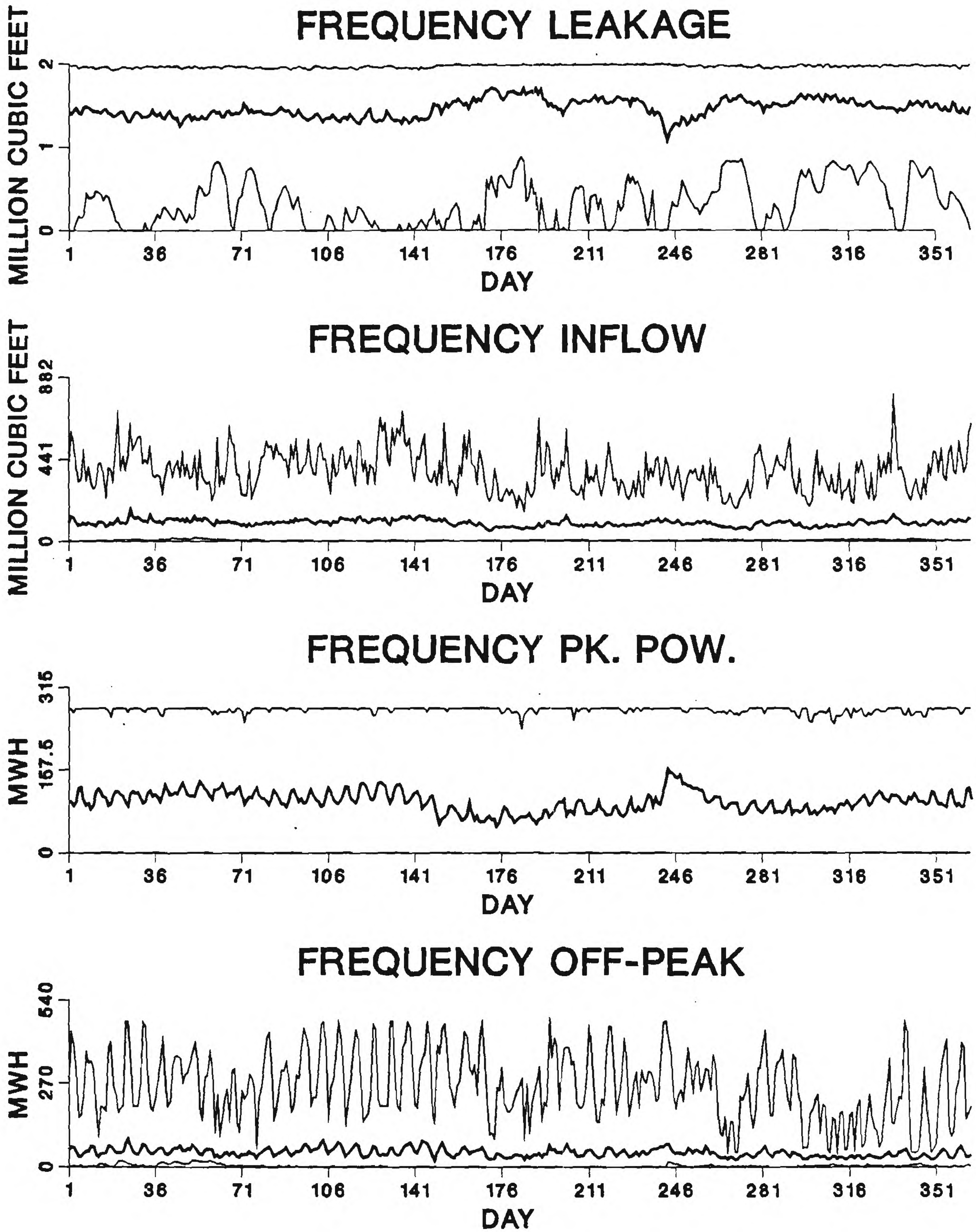


Figure 42: Daily Frequencies — Experiment V

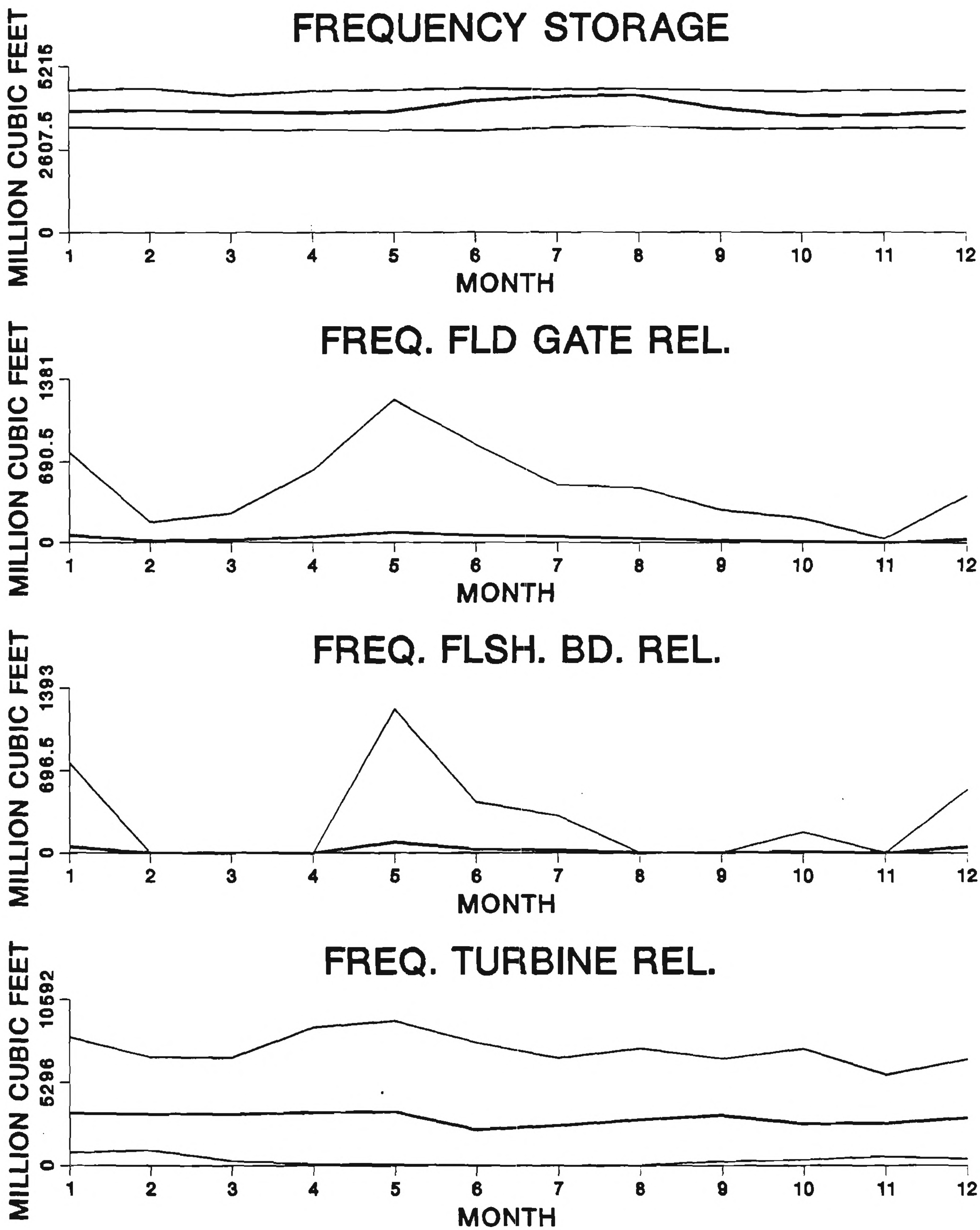
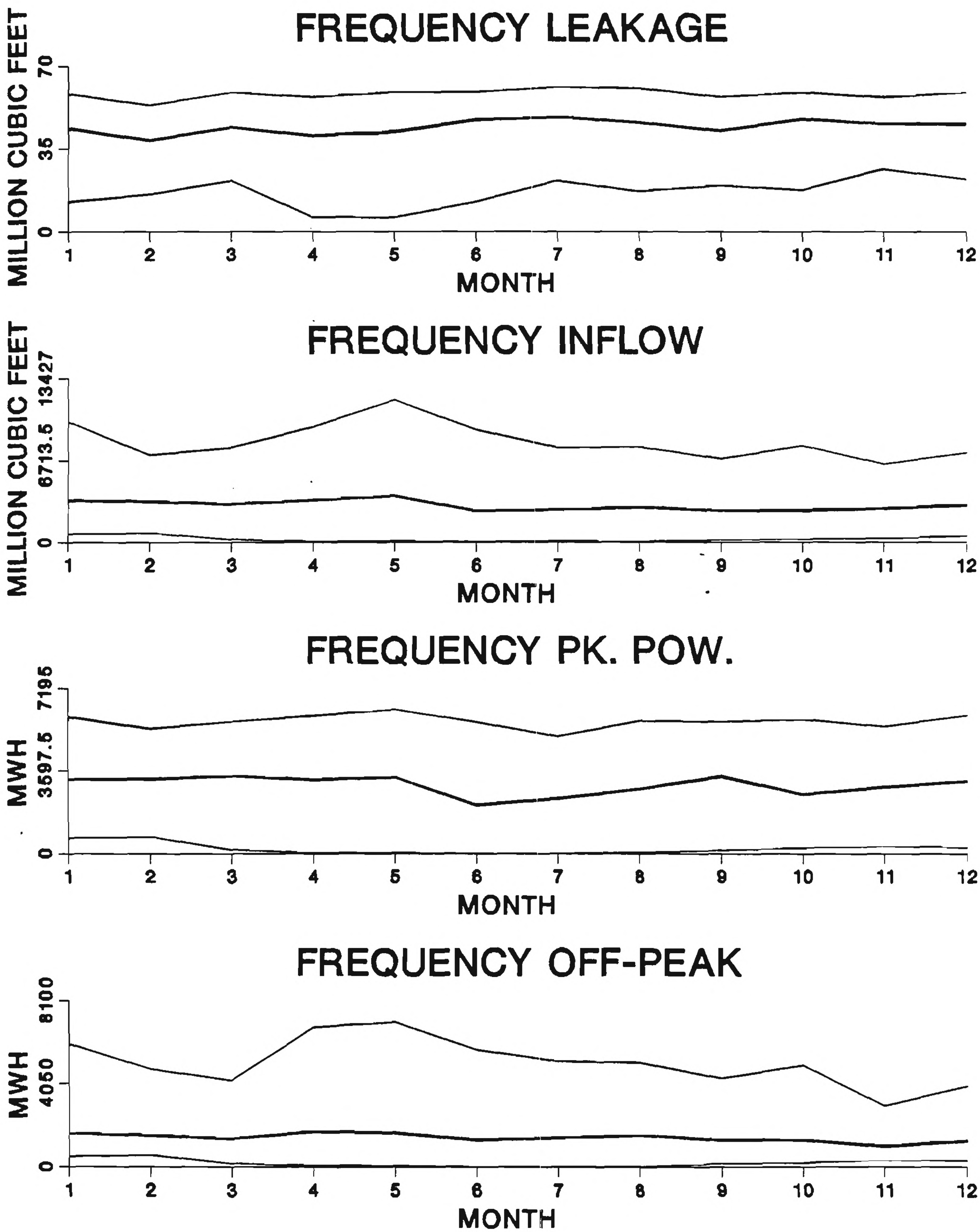


Figure 43: Monthly Frequencies — Experiment V



<u>Figure 44:</u> Monthly Frequencies — Experiment V

6. Concluding Remarks

This report discusses the theory and application of a new control model for the operation of the Lloyd Shoals hydroelectric facility. This model is based on a state-of-the-art stochastic control approach but also includes new enhancements that make it suitable for hydropower systems. The new model is designed to determine optimal power generation schedules on a daily basis and can also be used in a simulation mode to investigate policy issues. The following discussion summarizes the conclusions from such investigations.

As a general comment, stochastic control methods are expected to outperform deterministic approaches, because they are based on a more pragmatic system model. More specifically, the Lloyd Shoals facility was seen in the previous section to generate on the average 3,500 MWH more energy per year under stochastic control guidance. This gain represents 1,000 MWH of peak and 2,500 MWH of off-peak energy generation, or about $85,000 of yearly saved expenditures.

Streamflow forecasting enhances reservoir management by extending the decision time which is available to the operator. Due to inadequate rainfall and streamflow data records, a reliable streamflow forecasting model for Lloyd Shoals cannot be calibrated. However, the potential gains from improving the instrumentation network and developing such a model were estimated to be substantial. On the average, as much as 4,100 MWH of additional energy may be produced, about 1,300 MWH of which would represent peak and 2,800 MWH off-peak energy generation. These energy gains would translate into an average of $100,000 yearly saved expenditures. In that regard, a telemetry rainfall and streamflow instrumentation network would be the most appropriate real time data collection system.

The addition of a 7th turbine will enhance the plant capacity to accommodate flood volumes without having to resort to flood gate operation. The simulation analysis indicates that approximately 800 MWH of previously off-peak energy generation per year becomes available as peak energy generation. Naturally, the question is whether this improvement outweighs the unit purchasing and installation outlay. On the other hand, any expenditures reconditioning the existing turbines (in the way of leakage reduction) will result in considerable hydropower gains. According to the results of the previous section, such improvements may result in 7,500 MWH of additional energy production, 5,250 MWH of which would represent peak energy generation and the remaining off-peak energy improvements. On the average, these gains would amount to about $300,000 to $350,000 of yearly saved expenditures.

It is noted that all previous estimates represent gains pertaining to individual improvements. Thus, the total expected gain from a stochastic control model with accurate streamflow forecasting and six rehabilitated turbines would be about 15,000 MWH per year, about half of which would represent peak energy generation. On the average, this energy gain would constitute $500,000 of saved expenditures per year. Furthermore, usage of these techniques in larger hydroelectric projects or systems of hydroelectric projects is expected to generate higher profits.

The control model researched in this work is characterized by high computational efficiency. Thus, although the original computer code has been developed on a main frame computer (CYBER 180/990), it can also be modified for microcomputer implementation. An extensive description of the control and simulation programs and their use is provided in a separate document (Georgakakos, 1989c).

References

Brater, E. F., and H. W. King, *Handbook of Hydraulics*, McGraw-Hill, New York, 1963.

Benjamin, J. R., and C. A. Cornell, *Probability, Statistics, and Decision for Civil Engineers*, McGraw-Hill, New York, 1970.

Eagleson, P. S., Climate, Soil, and Vegetation: 2. The Distribution of the Annual Precipitation Derived From Observed Storm Sequences, *Water resources Research*, 14(5), 713-721, 1978.

Georgakakos, A. P., and D. H. Marks, A New Method for the Real Time Operation of Reservoir Systems, *Water Resources Research*, 23(7), 1376-1390, 1987.

Georgakakos, A. P., Extended Linear Quadratic Gaussian (ELQG) Control: Further Extensions, *Water Resources Research*, 25(2), 191-201, 1989a.

Georgakakos, A. P., The Value of Forecasting in Reservoir Control, in press, *Water Resources Bulletin*, 1989b.

Georgakakos, A. P., and J. C. Kabouris, A Streamflow Model Based on Geomorphologic Unit Hydrographs, in press, *Journal of Hydrology*, 1989.

Georgakakos, A. P., The GPOWER Control Program: User's Manual, *Technical Report*, Georgia Power Company under Research Project POE-06608, 1989c.

Georgia Power Company, Communication with Mr. Marvin Meeks, Section Supervising Engineer, 1988.

Marino, A. M., and A. H. Loaiciga, Dynamic Model for Multireservoir Operation, *Water Resources Research*, 21(5), 619-630, 1985.

Trezos, T. and W. W-G. Yeh, Use of Stochastic Dynamic Programming for Reservoir Management, *Water Resources Research*, 23(6), 983-996, 1987.

Wasimi, S. A., and P. K. Kitanidis, Real-time Forecast and Daily Operation of a Multireservoir System During Floods by Linear Quadratic Gaussian Control, *Water Resources Research*, 19(6), 1511-1522, 1983.